CONTEMPORARY MATHEMATICS

401

Snowbird Lectures on String Geometry

Proceedings of an AMS-IMS-SIAM
Joint Summer Research Conference
on String Geometry
June 5–11, 2004
Snowbird, Utah

Katrin Becker
Melanie Becker
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Contents

Preface

The interaction and cross-fertilization of mathematics and physics is ubiquitous in the history of both disciplines. However, the recent developments of string theory have led to some relatively new areas of common interest among mathematicians and physicists, and it was to explore some of these that the Snowbird conference of June 2004 was convened. Although not all papers presented were available for inclusion in this volume, the papers included provide a reasonably comprehensive sampling of the potential for fruitful interaction between mathematicians and physicists that exists as a result of string theory.

Much of the recent wave of interaction has been, on the mathematical side, in the area of algebraic geometry. There are two principal reasons for this: world sheets in string theory (at least in the Euclidean domain) are Riemann surfaces, and Calabi-Yau manifolds have been primary candidate vacua since the appearance of the paper of Candelas, Horowitz, Strominger and Witten on that subject. These two strands come together in the computation of instanton corrections in string theory, and lead to enumeration problems that are of interest to mathematicians and physicists alike. Perhaps the most obvious manifestation of this are the Gromov-Witten invariants, which are defined in terms of such enumerations and demonstrate the commonality of interest by their very name. The paper of Katz provides a nice review of the Gromov-Witten invariants and the related Gopakumar-Vafa and Donaldson-Thomas invariants. It goes on to present specific conjectural formulae relating the three sets of invariants and assembles evidence for the validity of the conjectures.

While it has been known for decades that some physically motivated constructions can be reformulated in terms of such mathematical concepts as sheaves, moduli and cohomology, the level of mathematical sophistication involved has increased markedly since the advent of string theory, especially in connection with the theory of D-branes. The concept Π-stability for BPS-brane configurations, with which Aspinwall's paper is concerned, is a case in point. Although motivated by the physical concept of stability against decay, it is defined in terms of the derived category of coherent sheaves on a Calabi-Yau manifold, but depends also on the complexified Kähler structure, regarded as a point in the moduli space of the mirror manifold. Aspinwall studies Π-stability for BPS-brane configurations supported on a Del Pezzo surface embedded in a Calabi-Yau manifold near the "wall" in the Kähler structure moduli space that corresponds to collapse of the Del Pezzo surface. In that regime, he is able to relate Π-stability to the more computable notion of θ-stability for quiver representations.

For some time now, there has been a consensus that toric varieties provide the appropriate context for the study of mirror symmetry from the mathematical point of view. The paper of Borisov and Horja (presented by Horja at the Snowbird conference) is concerned with Toric Deligne-Mumford stacks, which generalize toric varieties, and coherent sheaves over them, which generalize D-brane configurations. Mirror symmetry also generalizes to this context, and the authors promise to apply their results in that direction in a future paper.

Sharpe's paper is concerned with $(0, 2)$ mirror symmetry, which generalizes ordinary ($(2, 2)$) mirror symmetry to relate pairs of Calabi-Yau manifolds, each equipped with a holomorphic vector bundle. The manifolds involved need not be a mirror pair in the ordinary sense, except in the special case that the vector bundles are the holomorphic tangent bundles, in which case $(0, 2)$ mirror symmetry reproduces the usual mirror symmetry. The paper computes quantum correlation functions in this context, expressed as integrals over a compactified moduli space of world sheet instantons, obtained from gauged linear sigma models.

Although Calabi-Yau manifolds are, by definition, Kähler manifolds, it has long been known that there are examples of complex threefolds that admit no Kähler structure but satisfy the condition of vanishing canonical bundle that distinguishes Calabi-Yau manifolds from other Kähler manifolds. Although it has been some years since non-Kähler manifolds made their first appearance in in the physics literature in the mid 1980's, there has recently been a marked increase in the level of interest in this area among both mathematicians and physicists. From the string-theoretic point of view, the motivation to study non-Kähler manifolds is provided by the inclusion of fluxes. Such flux compactifications are of interest, as the inclusion of fluxes (i.e. non-vanishing tensor fields) in the background geometry lead to a potential for the moduli fields. String theorists hope that this will lead to a solution to the moduli space problems, so that predictions for the coupling constants for the standard model of elementary particles can be made in the future. From the mathematical point of view, such new background geometries are of interest, as many theorems of algebraic geometry need to be revisited in the context of non-Kähler manifolds with torsion. The papers of Krause and Tatar are examples of this relatively new trend.

At the time of the Snowbird conference, the revival of Twistor theory in the context of String theory as Twistor-String theory was just getting underway, exploiting ideas proposed by Witten. The paper of Roiban, Spradlin and Volovich, presented at the conference by Spradlin, follows Witten's prescription to compute scattering amplitudes for gluons in Yang-Mills theory. One of the features of the computation is that it involves integration over moduli spaces of instantons whose target is a complex supermanifold for which a super analogue of the Calabi-Yau condition holds, although the underlying manifold is not Calabi-Yau. Since the discovery of the AdS/CFT correspondence a number of interesting new results for large N strongly coupled Yang-Mills theories have been obtained in terms of the weakly coupled supergravity dual. Many times, there is an interesting mathematical structure behind such gauge theories, which has lead string theorists to believe they might be exactly solvable. This aspect has been emphasized in the paper by R.Roiban, who in his talk at the conference pointed out the existence of a relation between $N = 4$ Yang-Mills theory and an integrable spin chain.

To summarize, many new an interesting relations between mathematics and string theory have appeared in recent times and some of these aspects have been discussed at this conference. It has been more than a year since the Snowbird conference on String Geometry took place and the interaction between both disciplines continues growing. Both mathematicians and physicists have greatly benefited from this and hope that in the future advances in physics will lead to even greater advances in mathematics and vice versa.

List of Participants

Following is a list of the speakers at the Snowbird conference with the titles of their talks. The starred titles correspond to papers that appear in this volume. We have provided preprint references for as many as possible of the others.

Paul Aspinwall, D-branes on del Pezzo surfaces*

Robert Bryant, Geometry of G2 and Spin(7) structures[1]

Philip Candelas, Arithmetic of Calabi-Yau manifolds[2]

Emanuel Diaconescu, Extremal transitions in Gromov-Witten theory[3]

Ron Donagi, The particle spectrum of heterotic compactifications[4]

Michael Douglas, Open problems in Calabi-Yau compactification of IIb string theory[5]

Daniel Freed, The C-field in M-theory[6]

Tamar Friedmann, GUTs and dualities from singular G_2-geometry[7]

Sergei Gukov, Perturbative Yang-Mills theory and moduli spaces of curves[8]

Paul Horja, Toric Deligne-Mumford stacks and homological mirror symmetry*

Sheldon Katz, GW, GV, DT Invariants on Calabi-Yau 3-folds*

Axel Krause, Flux Compactification Geometries and de Sitter in Heterotic M-Theory*

Radu Roiban, On spin chains and field theories*

Eric Sharpe, Notes on correlation functions in (0,2) theories*

Marcus Spradlin, Yang-Mills Amplitudes from String Theory in Twistor Space*

Andrew Strominger, Black hole attractors and the topological string[9]

Radu Tatar, Geometric Transitions, Flops and Non-Kahler Manifolds*

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[1] math.DG/0305124

[2] hep-th/0012233, hep-th/0402133

[3] hep-th/0302076

[4] hep-th/0405014

[5] hep-th/0303194, hep-th/0307049, math.CV/0402326, hep-th/0404116

[6] hep-th/0409135

[7] hep-th/0203256, hep-th/0211269

[8] hep-th/0404085

[9] hep-th/0405146

Titles in This Series

401 **Katrin Becker, Melanie Becker, Aaron Bertram, Paul S. Green, and Benjamin McKay, Editors,** Snowbird lectures on string geometry, 2006

400 **Shiferaw Berhanu, Hua Chen, Jorge Hounie, Xiaojun Huang, Sheng-Li Tan, and Stephen S.-T. Yau, Editors,** Recent progress on some problems in several complex variables and partial differential equations, 2006

399 **Dominique Arlettaz and Kathryn Hess, Editors,** An Alpine anthology of homotopy theory, 2006

398 **Jay Jorgenson and Lynne Walling, Editors,** The ubiquitous heat kernel, 2006

397 **José M. Muñoz Porras, Sorin Popescu, and Rubí E. Rodríguez, Editors,** The geometry of Riemann surfaces and Abelian varieties, 2006

396 **Robert L. Devaney and Linda Keen, Editors,** Complex dynamics: Twenty-five years after the appearance of the Mandelbrot set, 2006

395 **Gary R. Jensen and Steven G. Krantz, Editors,** 150 Years of Mathematics at Washington University in St. Louis, 2006

394 **Rostislav Grigorchuk, Michael Mihalik, Mark Sapir, and Zoran Šuniḱ, Editors,** Topological and asymptotic aspects of group theory, 2006

393 **Alec L. Matheson, Michael I. Stessin, and Richard M. Timoney, Editors,** Recent advances in operator-related function theory, 2006

392 **Stephen Berman, Brian Parshall, Leonard Scott, and Weiqiang Wang, Editors,** Infinite-dimensional aspects of representation theory and applications, 2005

391 **Jürgen Fuchs, Jouko Mickelsson, Grigori Rozenblioum, Alexander Stolin, and Anders Westerberg, Editors,** Noncommutative geometry and representation theory in mathematical physics, 2005

390 **Sudhir Ghorpade, Hema Srinivasan, and Jugal Verma, Editors,** Commutative algebra and algebraic geometry, 2005

389 **James Eells, Etienne Ghys, Mikhail Lyubich, Jacob Palis, and José Seade, Editors,** Geometry and dynamics, 2005

388 **Ravi Vakil, Editor,** Snowbird lectures in algebraic geometry, 2005

387 **Michael Entov, Yehuda Pinchover, and Michah Sageev, Editors,** Geometry, spectral theory, groups, and dynamics, 2005

386 **Yasuyuki Kachi, S. B. Mulay, and Pavlos Tzermias, Editors,** Recent progress in arithmetic and algebraic geometry, 2005

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378 **Alexandre V. Borovik, Editor,** Groups, languages, algorithms, 2005

377 **G. L. Litvinov and V. P. Maslov, Editors,** Idempotent mathematics and mathematical physics, 2005

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The interaction and cross-fertilization of mathematics and physics is ubiquitous in the history of both disciplines. In particular, the recent developments of string theory have led to some relatively new areas of common interest among mathematicians and physicists, some of which are explored in the papers in this volume. These papers provide a reasonably comprehensive sampling of the potential for fruitful interaction between mathematicians and physicists that exists as a result of string theory.

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