

CONTEMPORARY MATHEMATICS

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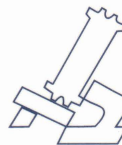
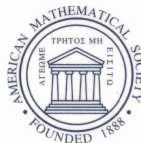
Israel Mathematical Conference Proceedings

Quantum Groups

Proceedings of a Conference
in Memory of Joseph Donin

July 5–12, 2004
Technion-Israel
Institute of Technology
Haifa, Israel

Pavel Etingof
Shlomo Gelaki
Steven Shnider
Editors



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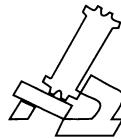
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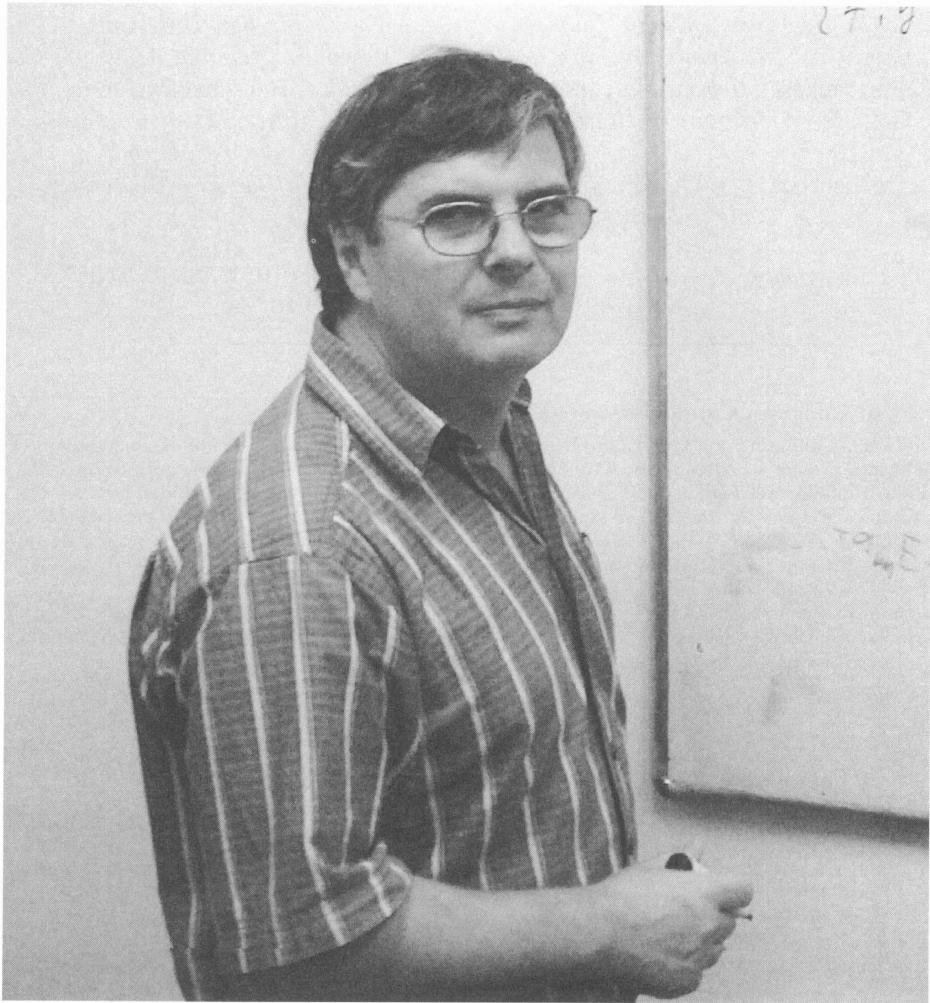


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Joseph Donin (1945–2004)

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2000 *Mathematics Subject Classification*. Primary 16W30, 81R50, 18D10, 17B37.

Library of Congress Cataloging-in-Publication Data

International Conference on Quantum Groups (2004 : Technion-Israel Institute of Technology)

Quantum groups : proceedings of a conference in memory of Joseph Donin, July 5–12, 2004, Technion, Haifa, Israel / [edited by] Pavel Etingof, Shlomo Gelaki, (Steven) Shnider.

p. cm. — (Israel mathematical conference proceedings) (Contemporary mathematics, ISSN 0271-4132 ; v. 433)

Includes bibliographical references.

ISBN 978-0-8218-3713-9 (alk. paper)

1. Quantum groups—Congresses. 2. Geometric quantization—Congresses. 3. Group theory—Congresses. I. Donin, Joseph, d. 2004. II. Etingof, P. I. (Pavel I.), 1969– III. Gelaki, Shlomo, 1964– IV. Shnider, S. (Steven), 1945– V. Title.

QA174.I557 2004

530.14'3—dc22

2007060761

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Contents

Preface	vii
Conference program	ix
List of participants	xiii
Joseph Donin's mathematical research, 1966–2003 A. MUDROV and S. SHNIDER	1
Geometric and unipotent crystals II: From unipotent bicrystals to crystal bases A. BERENSTEIN and D. KAZHDAN	13
The small quantum group and the Springer resolution R. BEZRUKAVNIKOV and A. LACHOWSKA	89
Relation between two geometrically defined bases in representations of GL_n A. BRAVERMAN, D. GAITSGORY, and M. VYBORNOV	103
Fourier transforms for Hopf algebras M. COHEN and S. WESTREICH	115
Quantization of some Poisson-Lie dynamical r -matrices and Poisson homogeneous spaces B. ENRIQUEZ, P. ETINGOF, and I. MARSHALL	135
Basic representations of quantum current algebras in higher genus B. ENRIQUEZ, S. PAKULIAK, and V. RUBTSOV	177
Poincaré-Birkhoff-Witt expansions of the canonical elliptic differential form G. FELDER, R. RIMÁNYI, and A. VARCHENKO	191
Geometry of non-commutative orbits related to Hecke symmetries D. GUREVICH and P. SAPANOV	209
Drinfeld double for orbifolds V. HINICH	251
Symmetrically factorizable groups and set-theoretical solutions of the pentagon equation R.M. KASHAEV and N. RESHETIKHIN	267
Dynamical reflection equation P.P. KULISH and A.I. MUDROV	281

On the combinatorics of Carter-Rieger-Saito movies in the theory of smoothly knotted surfaces in \mathbb{R}^4

G. LANCASTER, R. LARSON, and J. TOWBER

311

Preface

The International Conference on Quantum Groups was held from July 5-12, 2004, at the Technion-Israel Institute of Technology, Haifa, Israel. The conference was sponsored by the Center for Mathematical Studies (CMS) under the auspices of the Mallat Family Fund for Research in Mathematics, and was organized by Eli Aljadeff (Technion), Shlomi Gelaki (Technion), Miriam Cohen (Ben-Gurion University of the Negev), Pavel Etingof (MIT), David Kazhdan (Hebrew University), and Steve Shnider (Bar-Ilan University). The list of 52 speakers from 13 countries includes some of the leading researchers in quantum groups, Hopf algebras, and quantum algebra in general. The papers in this volume, whose topics sometimes differ from that of the lecture given by the author at the conference, have all been carefully refereed and represent significant contributions to their respective fields.

Originally, Joseph Donin was one of the invited speakers. Joseph's broad knowledge and deep understanding, together with his unassuming and congenial personality, made him a stimulating colleague to all those who knew him and worked with him. His sudden death on January 4, 2004, in the course of a routine medical procedure was a great loss to the mathematical community and, in particular, to those involved in research on deformation theory and quantization. Shortly thereafter, the organizers decided to dedicate the conference and the proceedings to his memory.

Joseph's first paper, "On deformations of holomorphic vector bundles over the Riemann sphere" was published in Math USSR Sbornik in 1966, while he was still working on his M.A. at Moscow State University. Two years later he published a much longer article based on material that would later appear in his Ph.D. thesis "On deformations of holomorphic fiber bundles over a compact complex space". He introduced a new approach which was based on the Banach manifold and Banach Lie group techniques of Douady and was completely different from the classical methods of Fischer and Grauert. Over the next ten years, he developed these ideas in a series of now classic papers on deformations of holomorphic bundles over complex spaces with singularities and versal deformations of germs of complex spaces.

From 1970 until immigrating to Israel in 1992, like many Jewish scientists in the former Soviet Union, Joseph worked at various research institutes: the Central Institute for Coal, the Central Institute for Industrial Building, and the Central Economics Mathematical Institute. During this period, in addition to continuing active research in pure mathematics, he published many technical reports in applied mathematics. Some of the topics in applied mathematics which he had investigated

continued to interest him. His last Ph.D. student was working on a project in applied mathematics connected to computer simulation of protein structures.

Since coming to Israel in 1992, Joseph's research interest focused on quantization, in particular, quantum groups and related questions in the quantization of R -matrices. A series of papers with D. Gurevich on the quantization of Poisson structures associated to R -matrices contained some of the first applications of quasi-Hopf algebras and the Drinfeld associator to quantization of homogeneous spaces. Over the next decade, he published over 20 papers on quantization problems, collaborating on various projects with Gurevich, Khoroshkin, Majid, Makar-Limanov, Mudrov, Rubtsov, and Shnider. The main topic of his research was quantizations invariant under the action of a quantum group, with a particular emphasis on the explicit description of the deformation of the defining relations. He and Gurevich proved the existence of such quantizations for flag varieties, orbits of the highest weight vector in irreducible representations and for semisimple coadjoint orbits. Using a combination of cohomological and quasi-Hopf methods, he and Shnider proved the existence of a two parameter invariant quantization on hermitian symmetric spaces. In a remarkable paper presented at the biannual Prague conference on quantum groups, Joseph proved the existence of a two parameter invariant quantization on the coadjoint representation of $sl(n)$. Joseph, together with Gurevich and Mudrov, extended the work of Kulish and his collaborators on the reflection equation algebra and constructed explicit invariant quantizations of orbits of $sl(n)$.

Joseph was a frequent visitor at the Max Planck Institute in Bonn and was always ready to consider new projects with the colleagues he met there. On one visit he worked with I. Penkov on inductive limits of algebraic varieties. On another visit he worked with P. Bressler on polarized deformation quantization. Recently, Joseph was involved in a flourishing research project with Andrey Mudrov. They introduced the concept of a base algebra and the associated dynamical category as a framework for studying the dynamical Yang-Baxter equation and unifying the work of Enriquez-Etingof and Xu. He died a month before his 59th birthday while preparing for a sabbatical at the MPI together with Andrey.

A more detailed account of Joseph's mathematical career is included in these proceedings.

May his memory live in the thoughts of all those who knew him.

The Editors

Conference Program

5-12 July, 2004

Monday, 5 July

- 08:30-09:30 Registration
- 09:30-09:40 Greetings and opening remarks

Morning Session

- 09:40-10:30 Olivier Schiffmann (Paris, France)
Canonical Bases of loop algebras via quot schemes
- 11:00-11:50 Alexander Braverman (Cambridge, Massachusetts)
Enumerative problems in algebraic geometry and quantum groups
- 12:00-12:50 Serguei Arkhipov (New Haven, Connecticut)
Semi-infinite Bernstein-Gelfand complex for small quantum groups and the global Cousin complex for the semi-infinite flag variety

Afternoon Session

- 14:30-15:20 Victor Ginzburg (Chicago, Illinois)
Quantum groups, the loop Grassmannian and the Springer resolution
- 15:30-16:20 Evgeny Mukhin (Indianapolis, Indiana)
Monomial bases in the Virasoro modules
- 16:50-17:40 Ivan Mirkovic (Boston, Massachusetts)
Localization for quantum groups at roots of unity

Tuesday, 6 July*Morning Session*

- 09:00-09:50 Jonathan Beck (Ramat-Gan, Israel)
Bases for quantum affine SL_2
- 10:00-10:50 Maxim Vybornov (Cambridge, Massachusetts)
Combinatorial perverse sheaves and IC-modules
- 11:20-12:10 Roman Bezrukavnikov (Evanston, Illinois)
Homological localization for quantum groups at root of unity

Afternoon Session

- 14:00-14:50 David Kazhdan (Jerusalem, Israel)
From geometric crystals to crystal bases
- 15:05-15:55 Dennis Gaitsgory (Chicago, Illinois)
Representations of the small quantum group and of affine algebras at the critical level
- 16:25-17:15 Sergei Loktev (Moscow, Russia)
Weyl modules over multi-dimensional currents
- 17:30-18:20 Amiram Braun (Haifa, Israel)
Quantum enveloping algebras and the unique factorisation property

Wednesday, 7 July*Morning Session*

- 09:00-09:50 Anthony Joseph (Rehovot, Israel)
Quantum KPRV determinants and their classical analogues
- 10:00-10:50 Steven Shnider (Ramat-Gan, Israel)
From equivariant quantization to dynamical Yang Baxter equations and back
- 11:20-12:10 Vladimir Roubtsov (Angers, France)
Elliptic algebras and commuting families in skew fields

Afternoon Session

- 14:00-14:50 Marc Rosso (Paris, France)
Tensor products of irreducible representations of quantum groups at a root of 1
- 15:05-15:55 Rinat Kedem (Urbana/Champaign, Illinois)
Fusion products and Kostka polynomials
- 16:25-17:15 Yuri Bazlov (London, U.K.)
"Bracket algebras" and noncommutative geometry of Weyl groups

Thursday, 8 July*Morning Session*

- 09:00-09:50 Victor Ostrik (Princeton, New Jersey)
Module categories over representations of quantum SL_2 and preprojective algebras
- 10:00-10:50 Hans Wenzl (La Jolla, California)
Braid representations related to spinors
- 11:20-12:10 Edward Effros (Los Angeles, California)
On the quantized functional analysis associated with quantum groups

Afternoon Session

- 14:00-14:50 Dmitri Nikshych (Durham, New Hampshire)
An analogue of Radford's S^4 formula for tensor categories
- 15:05-15:55 Yasuyuki Kawahigashi (Tokyo, Japan)
Operator algebras and vertex operator algebras
- 16:25-17:15 Nicolai Reshetikhin (Berkeley, California)
Braiding in quantum groups at roots of 1
- 17:30-18:20 David Evans (Cardiff, Wales)
Modular invariants and operator algebras

Friday, 9 July*Morning Session*

- 09:00-09:50 Hans-Juergen Schneider (Munich, Germany)
Representations of pointed Hopf algebras
- 10:00-10:50 Yorck Sommerhaeuser (Munich, Germany)
The index of a character
- 11:10-12:00 Yevgenia Kashina (Chicago, Illinois)
Higher Frobenius-Schur indicators for semisimple Hopf algebras

Afternoon Session

- 13:00-13:50 Stefaan Caenepeel (Brussels, Belgium)
The Brauer group of Azumaya corings and the second étale cohomology group
- 14:00-14:50 Earl Taft (Piscataway, New Jersey)
The search for a one-sided quantum group
- 15:10-16:00 Serban Raianu (Dominguez Hills, California)
Duality for Hopf algebras and corings

Sunday, 11 July*Morning Session*

- 09:00-09:50 Jasper Stokman (Amsterdam, The Netherlands)
Vector valued spherical functions on quantum groups
- 10:00-10:50 Erik Koelink (Delft, The Netherlands)
Dynamical quantum groups and special functions
- 11:20-12:10 Andrey Mudrov (Bonn, Germany)
Dynamical reflection equation

Afternoon Session

- 14:00-14:50 Jacob Greenstein (Geneva, Switzerland)
Filtrations and completions of certain positive level modules over quantum affine algebras
- 15:05-15:55 Arkady Berenstein (Eugene, Oregon)
Quantum cluster algebras
- 16:25-17:15 Dimitri Gurevich (Valenciennes, France)
Geometry of non-commutative orbits
- 17:30-18:20 Paul Bressler (Tucson, Arizona)
Levels and characters

Monday, 12 July*Morning Session*

- 09:00-09:50 Sonia Natale (Cordoba, Argentina)
Double categories and quantum groupoids
- 10:00-10:50 Jacob Towber (Chicago, Illinois)
Writhe- and amplitude-invariants of smooth 2-knots in R^4
- 11:20-12:10 Miriam Cohen (Beer Sheva, Israel)
Fourier transforms on Hopf algebras

Afternoon Session

- 14:00-14:50 Istvan Heckenberger (Rehovot, Israel)
Finite dimensional rank 2 Nichols algebras of diagonal type
- 15:00-15:50 Sara Westreich (Ramat-Gan, Israel)
Finite-dimensional pointed Hopf algebras of type A_n
- 16:10-17:00 Juan Jose Guccione (Buenos Aires, Argentina)
Hochschild homology of Frobenius algebras, with examples from Hopf algebra theory
- 17:10-18:00 Ian Marshall (Lausanne, Switzerland)
The momentum map for hidden symmetries of the Wess-Zumino-Novikov-Witten model

List of Participants

Eli Aljadeff Technion	Victor Ginzburg University of Chicago
Serguei Arkhipov Yale University	Jacob Greenstein University of California at Riverside
Yuri Bazlov University of Warwick	Jorge Guccione University of Buenos Aires
Jonathan Beck Bar-Ilan University	Dimitri Gurevich Université de Valenciennes, France
Arkady Berenstein University of Oregon	Istvan Heckenberger University of Leipzig
Roman Bezrukavnikov M.I.T.	Tim Hodges University of Cincinnati
Amiram Braun Haifa University	Anthony Joseph Weizmann Institute
Alexander Braverman Brown University	Yevgenia Kashina DePaul University, Chicago
Stefaan Caenepeel Virje University, Brussels	David Kazhdan Hebrew University of Jerusalem
Miriam Cohen Ben-Gurion University of the Negev	Rinat Kedem University of Illinois at Urbana/Champaign
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Pavel Etingof Massachusetts Institute of Technology	Erik Koelink Technical University, Delft
David Evans Cardiff University, Wales	Sergei Loktev Independent University of Moscow
Dennis Gaitsgory Harvard University	Ian Marshall EPFL, Lausanne, Switzerland
Shlomo Gelaki Technion	Ivan Mirkovic University of Massachusetts

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York University

Evgeny Mukhin
University of Indianapolis

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Argentina

Dmitri Nikshych
University of New Hampshire, Durham

Victor Ostrik
University of Oregon

Serban Raianu
California State University

Nicolai Reshetikhin
University of California, Berkeley

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Ecole Normale Superieure, Paris

Olivier Schiffmann
Ecole Normale Superieure, Paris

Hans-Juergen Schneider
University of Munich

Steven Shnider
Bar-Ilan University

Yorck Sommerhaeuser
University of Munich

Jasper Stokman
University of Amsterdam

Earl Taft
Rutgers University, Piscataway

Jacob Towber
DePaul University, Chicago

Maxim Vybornov
Massachusetts Institute of Technology

Hans Wenzl
University of California at San Diego

Sara Westreich
Bar-Ilan University

The papers in this volume are based on the talks given at the conference on quantum groups dedicated to the memory of Joseph Donin, which was held at the Technion Institute, Haifa, Israel in July 2004. A survey of Donin's distinguished mathematical career is included. Several articles, which were directly influenced by the research of Donin and his colleagues, deal with invariant quantization, dynamical R -matrices, Poisson homogeneous spaces, and reflection equation algebras. The topics of other articles include Hecke symmetries, orbifolds, set-theoretic solutions to the pentagon equations, representations of quantum current algebras, unipotent crystals, the Springer resolution, the Fourier transform on Hopf algebras, and, as a change of pace, the combinatorics of smoothly knotted surfaces.

The articles all contain important new contributions to their respective areas and will be of great interest to graduate students and research mathematicians interested in Hopf algebras, quantum groups, and applications.

ISBN 978-0-8218-3713-9



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