

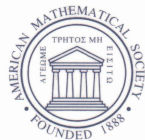
# CONTEMPORARY MATHEMATICS

449

## Group Representations, Ergodic Theory, and Mathematical Physics: A Tribute to George W. Mackey

AMS Special Session Honoring the Memory  
of George W. Mackey  
January 7–8, 2007  
New Orleans, Louisiana

Robert S. Doran  
Calvin C. Moore  
Robert J. Zimmer  
Editors



Group Representations, Ergodic  
Theory, and Mathematical Physics:  
A Tribute to George W. Mackey

Photo courtesy of Sigurdur Helgason.



George W. Mackey

1916–2006

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To

David F. Addis

with deep appreciation.

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# Preface

George W. Mackey was an extraordinary mathematician of great power and vision. His deep and profound contributions to representation theory, harmonic analysis, ergodic theory, mathematical physics, and related areas has left a rich legacy for researchers that continues today.

This volume contains the proceedings of an AMS Special Session entitled *Group Representations, Ergodic Theory, and Mathematical Physics: A Special Session Honoring the Memory of George W. Mackey* held on January 7-8, 2007 in New Orleans, Louisiana. The Table of Contents reveals contributions by an outstanding group of internationally known mathematicians and mathematical physicists. The papers range from expository and historical surveys to original research articles. The influence of George Mackey's fundamental ideas is apparent throughout these papers as well as the high esteem in which he is held by the authors. All of these articles, which span a rather broad range of mathematics, have been carefully refereed and will not appear elsewhere. The lead article entitled *George Mackey, 1916-2006*, is a slightly altered reprint of an article first published in the August 2007 issue of the Notices of the AMS. The text remains unchanged; however, a few of the photographs are new or have been repositioned. We thank the Notices editors for permission to reprint the article here.

Most of the papers that appear herein are expanded versions of the authors' talks in New Orleans and were written especially for this volume. Several are major surveys that took a great deal of time, effort, and commitment on the part of the authors. It was decided by the editors, after some deliberation, to present the papers in alphabetical order by author.

The editors express their sincere gratitude and thanks to all who made this volume possible. In particular, they wish to thank the other speakers for their beautiful presentations and their willingness to spend many hours writing them up so the results would be available to the larger mathematical community. We also acknowledge the hard work and help of the referees. We are especially indebted to David Addis, our technical associate, for a thoroughly professional job of getting the manuscript ready for publication. His great care for detail is reflected throughout this volume despite severe eye problems that would have caused a lesser person to quit. The volume is dedicated to Dave for his heroic efforts. We are deeply grateful to Ann Mackey for providing photos and for her support in many other ways. We thank Bruce Doran for carefully proofreading the entire manuscript and spotting many previously undetected errors and misprints. Finally, we wish to thank Sergei Gelfand, Christine Thivierge, Sandy Frost, Barbara Beeton, and others on the staff of the American Mathematical Society for their gracious help in publishing these proceedings.

Robert S. Doran

Calvin C. Moore

Robert J. Zimmer

George Mackey was an extraordinary mathematician of great power and vision. His profound contributions to representation theory, harmonic analysis, ergodic theory, and mathematical physics left a rich legacy for researchers that continues today. This book is based on lectures presented at an AMS special session held in January 2007 in New Orleans dedicated to his memory. The papers, written especially for this volume by internationally-known mathematicians and mathematical physicists, range from expository and historical surveys to original high-level research articles. The influence of Mackey's fundamental ideas is apparent throughout. The introductory article contains recollections from former students, friends, colleagues, and family as well as a biography describing his distinguished career as a mathematician at Harvard, where he held the Landon D. Clay Professorship of Mathematics.

Topics examined here include recent results on induced representations, virtual groups, the Mackey Machine and crossed products, representations of Baumslag-Solitar groups, the Radon transform and the heat equation, groupoids in the study of wavelets, and quantum theory. The in-depth historical surveys of Mackey's work on representation theory, ergodic theory, and physics, together with recent developments inspired by his fundamental work will be of considerable interest to both graduate students and researchers alike.

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