# CONTEMPORARY MATHEMATICS

### 450

## Poisson Geometry in Mathematics and Physics

International Conference June 5–9, 2006 Tokyo, Japan

> Giuseppe Dito Jiang-Hua Lu Yoshiaki Maeda Alan Weinstein Editors



American Mathematical Society

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American Mathematical Society Providence, Rhode Island

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#### Foreword

The conference "Poisson 2006: Poisson Geometry in Mathematics and Physics" was held from June 5 through 9, 2006 at the National Olympics Memorial Youth Center in Tokyo. There were about 150 participants, including 25 invited speakers, and 20 presenters at a poster session.

The speakers were chosen by a Scientific Committee of ten members, chaired by Alan Weinstein, while local organization was handled by a separate committee headed by Yoshiaki Maeda and Giuseppe Dito.

The meeting was preceded by a school of about three days, organized by Giuseppe Dito, Yoshiaki Maeda and Alan Weinstein, consisting of a lecture series designed to provide background for the conference talks, as well as invited topical lectures by young participants.

Sponsoring organizations for the conference and school included the Mathematical Society of Japan, the European Mathematical Society, the American Mathematical Society, the Bernoulli Center at EPFL Lausanne, and the 21st Century Center of Excellence (COE) at Keio University. The COE provided the majority of funding, with additional support from the US National Science Foundation.

Poisson 2006 was the fifth in a series of international conferences on Poisson geometry, held every two years. The first, in 1998, took place at the Banach Centre in Warsaw, with subsequent meetings at CIRM in Luminy, IST in Lisbon, and the University of Luxembourg. Further information about all these meetings, as well as the one to be held in 2008 at EPFL in Lausanne, may be found on the Poisson Geometry Home Page at poissongeometry.org, which links to the videos of all the talks of the conference Poisson 2006 and principal lectures of the school.

The aim of these meetings has been to bring together mathematicians and mathematical physicists who work in diverse areas and share a common interest in Poisson geometry. With roots in classical mechanics from 200 years ago and the work of Sophus Lie from a century ago, the subject of Poisson geometry crystallized through the work of Kirillov and Lichnerowicz in the 1970's and has been particularly driven by the program of "deformation quantization", in which Poisson structures appear as the first deviation from commutativity in families of associative algebras. Subjects where Poisson geometry plays an essential role include symplectic geometry and topology, deformation theory, representation theory, hamiltonian dynamics, and field theory.

In preparing the program for Poisson 2006, the Scientific Committee made a special effort to include speakers from "outside" areas which were relevant to Poisson geometry and its applications. The program of Poisson 2006 (conference and school) was remarkable for the overlap of topics, some intentional, some fortuitous, between the lectures. Here are some examples:

#### FOREWORD

- Generalized complex stuctures and other geometry on  $TM + T^*M$  (Bursztyn, Gualtieri, Meinrenken, Uchino, Yoshimura)
- Stacks and twisting by a three-form (Gomi, Schapira, Tsygan, Van den Bergh)
- Orbifolds and other singular spaces as differentiable stacks (Crainic, Holm, Weinstein, Xu, Zhu)
- Normal forms of Poisson structures in the neighborhood of points and symplectic leaves (Dufour, Fernandes, Ratiu, Zung)
- Deformation of Poisson structures (Ikeda, Zhang)
- Reduction of systems with symmetry (Bursztyn, Cardona, Cattaneo, Holm, Ratiu, Yoshimura)
- Kontsevich formality and its variants (Alekseev, Kontsevich, Merkulov, Park, Tsygan, Van den Bergh, Waldmann)
- Log-canonical coordinates (Gekhtman, Kontsevich, Lu)
- Group-valued momentum maps (Meinrenken, Schaffhauser)
- Strict quantization of spaces via group actions (Rieffel, Voglaire, Waldmann)
- Quantization of canonical transformations via their graphs (Kontsevich, Schapira, Van den Bergh, Weinstein)

The present volume consists of refereed papers by many of the invited speakers at the conference and by the principal lecturers at the school. Papers by presenters at the poster session and by other speakers at the school will be appearing in *Travaux Mathématiques*.

Giuseppe Dito, Jiang-Hua Lu, Yoshiaki Maeda, Alan Weinstein

#### List of Participants

Alekseev, Anton Université de Genève, Switzerland

Androulidakis, Iakovos Universität Zürich, Switzerland

Antunes, Paulo École Polytechnique, France

Arias Abad, Camilo Universiteit Utrecht, The Netherlands

Baird, Tom University of Toronto, Canada

Bialecki, Mariusz University of Tokyo, Japan and Academy of Sciences, Poland

Blohmann, Christian University of California at Berkeley, USA

Bonneau, Philippe Université de Bourgogne, France

Brahami, Renaud Université de Bourgogne, France

Brahic, Olivier Instituto Superior Técnico, Portugal

Bursztyn, Henrique Instituto Nacional de Matemática Pura e Aplicada, Brazil

Cardona, Alexander Universidad de Los Andes, Colombia

Cattaneo, Alberto Universität Zürich, Switzerland

Chen, Zhuo Capital Normal University, China Chiang, River National Cheng Kung University, Taiwan

Claessens, Laurent Université catholique de Louvain, Belgium

Crainic, Marius Universiteit Utrecht, The Netherlands

Dherin, Benoit Université de Genève, Switzerland

Dinar, Yassir International School for Advanced Studies (SISSA), Italy

Dito, Giuseppe Université de Bourgogne, France

Dragulete, Oana Ecole Polytechnique Fédérale de Lausanne, Switzerland

Dubrovskiy, Stanislav Keio University, Japan

Dufour, Jean-Paul Université de Montpellier 2, France

Fernandes, Rui Instituto Superior Técnico, Portugal

Fregier, Yael University of Luxembourg, Grand-Duchy of Luxembourg

Fuchs, Shay University of Toronto, Canada

Fujita, Daisuke Keio University, Japan Futaki, Akito Tokyo Institute of Technology, Japan

Futaki, Masahiro University of Tokyo, Japan

Gay-Balmaz, François Ecole Polytechnique Fédérale de Lausanne, Switzerland

Gekhtman, Michael University of Notre Dame, USA

George, Nathan University of California at Berkeley, USA

Ginzburg, Viktor University of California at Santa Cruz, USA

Gomi, Kiyonori University of Tokyo, Japan

Grabowski, Janusz Polish Academy of Sciences, Poland

Gualtieri, Marco Massachusetts Institute of Technology, USA

Hamachi, Kentaro Kyoto Sangyo University, Japan

Hamilton, Mark University of Calgary, Canada

Harada, Megumi University of Toronto, Canada

Haruyama, Daisuke Keio University, Japan

Hayakawa, Yohei Keio University, Japan

He, Long-Guang Capital Normal University, China

Hirota, Yuji Keio University, Japan

Ho, Nan-Kuo The Fields Institute, Canada

Hofer, Laurent Université de Haute-Alsace, France Holm, Tara University of Connecticut, USA

Iglesias Ponte, David Instituto de Matemáticas y Física Fundamental, Spain

Ikeda, Kaoru Keio University, Japan

Ikeda, Noriaki Ritsumeikan University, Japan

Indelicato, Davide Universität Zürich, Switzerland

Inoue, Rei University of Tokyo, Japan

Ito, Yuji Keio University, Japan

Iwata, Etsuji Waseda University,Japan

Jane, James University of Cambridge, United Kingdom

Kajiura, Hiroshige Kyoto University, Japan

Kameta, Keisei Institute of Physics Publishing, Japan

Kato, Daisuke Keio University, Japan

Keller, Frank Université du Luxembourg, Grand-Duchy of Luxembourg

Kieserman, Noah University of Winsconsin, USA

Kim, Donghui Yonsei University, Korea

Kim, Hoil Kyungpook National University, Korea

Kimura Takashi Boston University, USA

Kirillov, Anatol RIMS, Kyoto University, Japan

х

#### PARTICIPANTS

Koda, Yuya Keio University, Japan

Kohmoto, Daichi Nagoya University, Japan

Konishi, Yukiko University of Tokyo, Japan

Kontsevich, Maxim Institut des Hautes études Scientifiques, France

Kori, Tosiaki Waseda University, Japan

Kosmann-Schwarzbach, Yvette Ecole Polytechnique, France

Kubarski, Jan Technical University of Lodz, Poland

Kubo, Fujio Hiroshima University, Japan

Larrain Hubach, Andres Universidad de los Andes, Colombia

Le, Thanh Hieu Quy Nhon University, Vietnam

Lee, Sungyun Korean Advanced Institute of Science and Technology, Korea

Li, Hui Instituto Superior Técnico, Portugal

Liu, Zhangju Beijing University, China

Lu, Jiang-Hua University of Hong Kong, Hong Kong

Lyapina, Oleksandra University of Notre Dame, USA

Machida, Yoshinori Numazu National college of Technology, Japan

Maeda, Yoshiaki Keio University, Japan

Martens, Johan University of Toronto, Canada Martinez Torres, David Utrecht University, The Netherlands

Mehta, Rajan University of California at Berkeley, USA

Meinrenken, Eckhard University of Toronto, Canada

Merkulov, Sergei Stockholms Universitet, Sweden

Mikami, Kentaro Akita University, Japan

Miranda, Eva Université de Toulouse 3, France

Miyake, Ken Keio University, Japan

Miyaoka, Reiko Kyushu University, Japan

Miyazaki, Naoya Keio University, Japan

Mizutani, Tadayoshi Saitama University, Japan

Monnier, Philippe Université de Toulouse 3, France

Moreau, Anne Université Paris 7, France

Moriyoshi, Hitoshi Keio University, Japan

Nakanishi, Nobutada Gifu-Keizai University, Japan

Nest, Ryszard Copenhagen University, Denmark

Nguyen, Minh Cong University of Education of Hanoi, Vietnam

Nguyen, Tien Zung Université de Toulouse 3, France

Nunes da Costa, Joana Universidade de Coimbra, Portugal

#### PARTICIPANTS

Oguni, Shin-ichi Kyoto University, Japan

Oikonomides, Catherine Keio University, Japan

Omori, Hideki Tokyo University of Science, Japan

Ono, Kaoru Hokkaido University, Japan

Otgonbayar, Uuye Pennsylvania State University, USA

Park, Jae-Suk Yonsei University, Korea

Park, Seo-Ree Seoul National University, Korea

Peterka, Mira University of California at Berkeley, USA

Pflaum, Markus Johann Wolfgang Goethe-Universität, Germany

Pichereau, Anne Université de Poitiers, France

Poncin, Norbert University of Luxembourg, Grand-Duchy of Luxembourg

Ratiu, Tudor École polytechnique fédérale de Lausanne, Switzerland

Raugas, Mark RABA Technologies, USA

Rieffel, Marc University of California at Berkeley, USA

Sako, Akifumi Keio University, Japan

Sato, Nobuya Rikkyo University, Japan

Schaetz, Florian Universität Zürich, Switzerland Schaffhauser, Florent Keio University, Japan

Schapira, Pierre Université de Paris 6, France

Schlichenmaier, Martin University of Luxembourg, Grand-Duchy of Luxembourg

Siby, Hassène Université de Montpellier 2, France

Signori, Daniele Pennsylvania State University, USA

Sjamaar, Reyer Cornell University, USA

Stefanini, Luca Universität Zürich, Switzerland

Sternheimer, Daniel Université de Bourgogne, France and Keio University, Japan

Stienon, Mathieu Pennsylvania State University, USA

Suzuki, Haruo Hokkaido University, Japan

Takhtajan, Leon SUNY at Stony Brook, USA

Tanaka, Yuuji Nagoya University, Japan

Terashima, Yuji Tokyo Institute of Technology, Japan

Terilla, John CUNY - Queens College, USA

Tomoda, Atsushi Keio University, Japan

Tose, Nobuyuki Keio University, Japan

Tsygan, Boris Northwestern University, USA

Tudoran, Razvan Micu Universitatea de Vest din Timisoara, Romania

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Uchino, Kyosuke Tokyo University of Science, Japan

Urbanski, Pawel University of Warsaw, Poland

Van den Bergh, Michel Universiteit Hasselt, Belgium

Voglaire, Yannick Université catholique de Louvain, Belgium

Wade, Aissa Pennsylvania State University, USA

Waldmann, Stefan Albert-Ludwigs-Universität Freiburg, Germany

Watamura, Satoshi Tohoku University, Japan

Weinstein, Alan University of California at Berkeley, USA

Xu, Ping Pennsylvania State University, USA

Yoshida, Takahiko University of Tokyo, Japan

Yoshimura, Hiroaki Waseda University, Japan

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Zambon, Marco Universität Zürich, Switzerland

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This volume is a collection of articles by speakers at the conference "Poisson 2006: Poisson Geometry in Mathematics and Physics", which was held June 5–9, 2006, in Tokyo, Japan. Poisson 2006 was the fifth in a series of international conferences on Poisson Geometry that are held once every two years. The aim of these conferences is to bring together mathematicians and mathematical physicists who work in diverse areas but have common interests in Poisson Geometry. The program for Poisson 2006 was remarkable for the overlap of topics that included deformation quantization, generalized complex structures, differentiable stacks, normal forms, and group-valued moment maps and reduction. The articles represent current research in Poisson Geometry and should be valuable to anyone interested in Poisson Geometry, symplectic geometry, and mathematical physics. This volume also contains lectures by the principal speakers of the three-day school held at Keio University that preceded Poisson 2006.



