

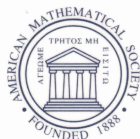
CONTEMPORARY MATHEMATICS

451

Frames and Operator Theory in Analysis and Signal Processing

AMS-SIAM Special Session
January 12–15, 2006
San Antonio, Texas

David R. Larson
Peter Massopust
Zuhair Nashed
Minh Chuong Nguyen
Manos Papadakis
Ahmed Zayed
Editors



Frames and Operator Theory in Analysis and Signal Processing

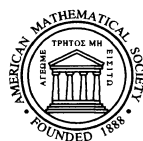
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2000 *Mathematics Subject Classification*. Primary 11S80, 20F55, 35S99, 41A30, 42C15, 42C40, 47N40, 47S10, 94A12.

Library of Congress Cataloging-in-Publication Data

Frames and operator theory in analysis and signal processing : AMS Special Session, January 12–15, 2006, San Antonio, Texas / David R. Larson...[et al.], editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; v. 451)

Includes bibliographical references.

ISBN 978-0-8218-4144-0 (alk. paper)

1. Operator theory—Congresses. 2. Signal processing—Congresses. I. Larson, David R., 1942–

QA329.F73 2008

515'.724—dc22

2007060586

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Preface

The collection of papers in this volume is based on presentations given at the AMS Special Session entitled *Frames and Operator Theory in Analysis and Signal Processing*, which took place at the Joint Mathematics Meetings in San Antonio, Texas, on January 12 – 15, 2006.

One of the goals of this special session was to integrate both the industrial and theoretical aspects of frames and operator theory, in particular their applications to image and signal processing. In addition, the special session contained a strong component of works related to sampling and numerical solutions of partial differential equations. The connections with operator theory (the pure and academic aspects of the special session) reflect a broader group than represented in the previous special sessions dealing with the afore-mentioned topics. Signal and image processing use common mathematical techniques; chief among them are functional analysis, inverse problems, applied harmonic analysis, including wavelets, and transform techniques.

During recent years, the field of frames has undergone a tremendous advancement. Most of the work in these fields has focused on the design and construction of more versatile frames and, more recently, frames tailored towards specific applications, e.g. finite dimensional uniform frames for cellular communication. In addition, frames are now becoming a new hot topic in mathematical research, but surprisingly enough it appears that this recently rediscovered topic is in the heart of many engineering applications, e.g. matching pursuits and greedy algorithms for image and signal processing.

Topics that the special session covered include:

- Application of several branches of analysis (e.g., PDEs, Fourier, wavelet, and harmonic analysis, transform techniques, data representations) to industrial and engineering problems, in particular image and signal processing.
- Theoretical and applied aspects of frames and wavelets.
- Pure aspects of operator theory emphasizing the connections to applied mathematics, frames, and signal processing.

The above-mentioned themes and their scope of applicability are reflected in the papers in this Conference Proceedings volume. A number of interesting and innovative presentations regarding frames and their interactions with operator theory are exhibited and several of these techniques have already been designed for a specific application while others seem to have a potential for future success in applications. For these reasons, we believe that putting together a series of semi-expository and original research papers with the flavor inferred from the above topics will produce a unique volume in the operator and frames literature. We also need to mention that

the interest in frames from both the mathematical and the engineering community has recently been rekindled. This renewed interest is mostly due to the foreseen immense potential for applications in the imaging sciences and the burgeoning area of mobile communication. Despite these developments, the current literature on frames and their relation to operators is rather limited and the monographs that are available contain the general theory or parts of it with only minor references to current research trends. A major weakness, in our opinion, of these publications is the lack of information about the applications of frame theory and their connections to operators. The current volume is the first of its kind to simultaneously address theoretical foundations of frame design and operator theory and elucidate their mutual applications.

We believe that this Conference Proceedings volume will be equally attractive to pure mathematicians, working on the foundations of frame and operator theory and their interconnections, to applied mathematicians, who are investigating applications, and to physicists and engineers employing these designs. It thus may be attractive to a wide target group of researchers and may serve as a catalyzer for cross-fertilization of several important areas of mathematics and the applied sciences.

David R. Larson, Peter Massopust, Zuhair Nashed,
Ming Chuong Nguyen, Manos Papadakis, and Ahmed Zayed
September 28, 2007

**Speakers and Titles of Talks presented in the AMS - SIAM
Special Session on Frames and Operator Theory in Analysis
and Signal Processing, in the Joint Mathematics Meetings,
San Antonio, Texas, January 12-15, 2006**

Radu V. Balan
Siemens Corporate Research
“A Noncommutative Wiener Lemma
and a Faithful Tracial State on Banach
Algebras of Time-Frequency Shift
Operators”

John J. Benedetto
University of Maryland
“Waveform Design and a General Form
of Matched Filtering”

Peter G. Casazza
University of Missouri
“The Kadison-Singer Problem in
Mathematics and Engineering: Part II”

Ingrid Daubechies
Princeton University
“Two Applications of Frames for the
Analysis of Astrophysical Data”

Maarten V. De Hoop
Purdue University
“Curvelets and the One-Way Acoustic
Wave Equation”

P.P.B. Eggermont
University of Delaware
“Tikhonov Regularization of Ill-posed
Operator Equations with Weakly
Bounded Noise”

Adel Faridani
Oregon State University
“Construction of Sampling Theorems
for Unions of Shifted Lattices”

Deguang Han
University of Central Florida
“Lattice Tilings, Operator Algebras
and Gabor Frames”

Palle Jorgenson
University of Iowa
“Use of Geometry and Operator
Algebra Theory in the Computation of
Wavelet Coefficients”

Victor Kaftal
University of Cincinnati
“Operator-valued Frames over Hilbert
C*-modules”

Costas Karanikas
Aristotle University of Thessaloniki,
Greece
“Haar-type Orthonormal Systems, Data
Presentation as Riesz Products and a
Prediction Method on Symbolic
Sequences”

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University of Houston
“New Approaches to the Acoustic Wave
Equation”

Mila Nikolova
CMLA UMR 8536–ENS de Cachan,
France
“Data Fidelity Terms for Image and
Signal Restoration”

Gestur Olafsson
Louisiana State University
“Wavelets, Multiresolution Analysis
and Finite Reflexion Groups”

Judith A. Packer
University of Colorado at Boulder
“The Use of Filters and Direct Limits in
the Construction of Fractal Wavelets”

Vern I. Paulsen
University of Houston
“Frame Paths and Sigma-Delta
Quantization”

Luigi Rodino
Università di Torino, Italy
“Pseudo-Differential Operators,
Localization Operators and
Time-Frequency Analysis”

M. Beth Ruskai
Tufts University
“The Relation between Frames and
POVM’s in Quantum Information
Theory”

Otmar Scherzer
Universität Innsbruck, Austria
“Regularization Functionals Involving
Discontinuous Operators”

Reinhold Schneider
Christina-Albrechts-Universität Kiel,
Germany
“Multiscale Approximation in
Electronic Structure Calculation”

Xiaopeng Shen
Ohio University
“Prolate Spheroidal Wavelets in a
Periodic Setting”

Hrvoje Šikić
University of Zagreb, Croatia
“The Structure of the Set of Parseval
Frame Wavelets”

Darrin Speegle
St. Louis University
“The Feichtinger Conjecture for Frames
of Translates”

Qiyu Sun
University of Central Florida
“Oversampling a Tight Affine Frame”

Gilbert Walter
University of Wisconsin-Milwaukee
“Multidimensional Prolate Spheroidal
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Yang Wang
Georgia Institute of Technology
“On PCM and Sigma-Delta
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Eric Weber
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ISBN 978-0-8218-4144-0



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