CONTEMPORARY MATHEMATICS

458

Integrable Systems and Random Matrices

In Honor of Percy Deift

Conference on Integrable Systems,
Random Matrices, and Applications
in Honor of Percy Deift's 60th Birthday
May 22–26, 2006
Courant Institute of Mathematical Sciences
New York University, New York

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Thomas Kriecherbauer
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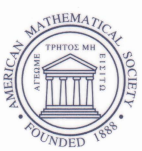
Editors



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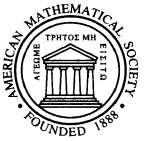

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With best wishes to

Percy A. Deift

on the occasion of his 60^{th} birthday.

Contents

Preface ix

List of Participants xi

Percy Deift at integer times
CARLOS TOMEI 1

Birkhoff normal form for the periodic Toda lattice
ANDREAS HENRICI and THOMAS KAPPELER 11

Asymptotics of a Class of Operator Determinants with Application to the Cylindrical Toda Equations
HAROLD WIDOM 31

Connection formulas for the third Painlevé transcendent in the complex plane
V. YU. NOVOKSHENOV 55

Nonlinear Fourier Transforms and Integrability in Multidimensions
A. S. FOKAS 71

Numerical study of a multiscale expansion of Korteweg-de Vries and Camassa-Holm equation
TAMARA GRAVA and CHRISTIAN KLEIN 81

Long-time asymptotics of the Camassa-Holm equation on the line
ANNE BOUTET DE MONVEL and DMITRY SHEPELSKY 99

Semiclassical Focusing Nonlinear Schrödinger equation in the pure radiation case: Riemann-Hilbert Problem approach
ALEXANDER TOVBIS, STEPHANOS VENAKIDES, and XIN ZHOU 117

From Stationary Phase to Steepest Descent
SPYRIDON KAMVISSIS 145

Riemann-Hilbert Problems with Lots of Discrete Spectrum
PETER D. MILLER 163

Locating the zeros of partial sums of e^z with Riemann-Hilbert methods
T. KRIECHERBAUER, A. B. J. KUIJLAARS, K. D. T-R MCLAUGHLIN, and P. D. MILLER 183

Two variable Deformations of the Chebyshev measure
JEFFREY S. GERONIMO and PLAMEN ILIEV 197

Hankel determinant and orthogonal polynomials for the Gaussian weight with a jump
A. ITS and I. KRASOVSKY 215

Orthogonal Polynomials, Jacobi Matrices and Random Matrices
L. PASTUR 249

Universality in unitary random matrix ensembles when the soft edge meets the hard edge
TOM CLAEYS and ARNO B.J. KUIJLAARS 265

A new approach to universality limits at the edge of the spectrum
D. S. LUBINSKY 281

On the gap probability generating function at the spectrum edge in the case of orthogonal symmetry
P. J. FORRESTER 291

An interpolation between Airy and Pearcey processes
MARK ADLER and PIERRE VAN MOERBEKE 303

The universal Airy_1 and Airy_2 processes in the Totally Asymmetric Simple Exclusion Process
PATRIK L. FERRARI 321

On some special directed last-passage percolation models
KURT JOHANSSON 333

Discrete spacetime and its applications
EITAN BACHMAT 347

The Arctic Circle Revisited
F. COLOMO and A. G. PRONKO 361

Moduli of continuity and average decay of Fourier transforms: two-sided estimates
DIMITRI GIOEV 377

Eigenvalues of Schrödinger Operators in Gaps of the Essential Spectrum — an Overview
RAINER HEMPEL 393

Transition Measures for the Stochastic Burgers Equation
JONATHAN C. MATTINGLY and TOUFIC M. SUIDAN 409

Some open problems in random matrix theory and the theory of integrable systems
PERCY DEIFT 419

Preface

This volume contains the proceedings of a conference on *Integrable Systems, Random Matrix Theory, and Applications* held in New York in May 2006. In addition to its scientific goals the conference served to celebrate the sixtieth birthday of Percy A. Deift and to honor his many profound contributions to mathematics.

The theory of integrable systems had its modern rebirth in the 1960's when the complete integrability of the Korteweg – de Vries equation was discovered and understood. Since that time, the theory has made many deep connections with diverse areas of mathematics and the theory of random matrices is one of them. A milestone in uncovering the beautiful relations between random matrices and integrable systems was the work of Baik, Deift and Johansson on the distribution of the length of the longest increasing subsequence of random permutations. Using the nonlinear steepest descent method for Riemann-Hilbert problems that was developed by Deift and Zhou in the context of integrable systems they found a surprising connection between random permutations and random matrices. This discovery triggered an enormous amount of research activities that continues to this day and that involves additional fields such as representation theory, orthogonal polynomials, spectral theory, operator theory, probability and statistics. It was the scientific purpose of this conference to bring researchers from these areas together in order to discuss recent developments as well as future directions of research.

The five-day conference attracted 100 participants from 15 countries. The program of the conference, audio and pdf files of most of the talks and a video of the Discussion Session can be found under *http://math.arizona.edu/~mcl/ISRMA.html.*

Finally, it is our happy task to thank those who made it all possible. We thank the American Institute of Mathematics and the Courant Institute of Mathematical Sciences for their support, both financial and infrastructural, which both got the ball rolling, and carried the conference through to its conclusion, and we thank the National Science Foundation for their support through grant NSF-DMS 0553069. It is also a pleasure to thank the staff of the Courant Institute for the excellent organization of the conference. Last (but certainly not least!) we express our gratitude to the participants of the conference who made it an memorable event, to the contributors to this volume and to Christine Thivierge and the AMS technical staff for their professional and most welcome help with editing these proceedings.

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This volume contains the proceedings of a conference held at the Courant Institute in 2006 to celebrate the 60th birthday of Percy A. Deift. The program reflected the wide-ranging contributions of Professor Deift to analysis with emphasis on recent developments in Random Matrix Theory and integrable systems. The articles in this volume present a broad view on the state of the art in these fields. Topics on random matrices include the distributions and stochastic processes associated with local eigenvalue statistics, as well as their appearance in combinatorial models such as TASEP, last passage percolation and tilings. The contributions in integrable systems mostly deal with focusing NLS, the Camassa–Holm equation and the Toda lattice. A number of papers are devoted to techniques that are used in both fields. These techniques are related to orthogonal polynomials, operator determinants, special functions, Riemann–Hilbert problems, direct and inverse spectral theory. Of special interest is the article of Percy Deift in which he discusses some open problems of Random Matrix Theory and the theory of integrable systems.

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