# CONTEMPORARY MATHEMATICS

### 464

# Radon Transforms, Geometry, and Wavelets

AMS Special Session January 7–8, 2007 New Orleans, Louisiana

Workshop January 4–5, 2007 Baton Rouge, Louisiana

Gestur Ólafsson Eric L. Grinberg David Larson Palle E. T. Jorgensen Peter R. Massopust Eric Todd Quinto Boris Rubin Editors



American Mathematical Society

## Radon Transforms, Geometry, and Wavelets

# CONTEMPORARY MATHEMATICS

359

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#### Preface

In recent years a number of specialties in mathematical analysis, geometry and number theory have come together. We have witnessed ideas from one field connecting with and being used in surprising ways in another fields, and making contacts to applications involving the Radon transforms, frames, wavelets, and fractals; and to applications outside of mathematics as well: engineering, physics, and medical imaging. This works in reverse as well, including even applications and basic ideas from signal processing to mathematics! This multifaceted enterprise has a common core: harmonic analysis and symmetry. The purpose of the present volume of *Contemporary Mathematics* is to present some of this fascinating material and connections in a form that is understandable and attractive to a wider readership, including graduate students in mathematics and its neighboring fields.

Harmonic analysis is related to-and inspired by-several branches of mathematics and science: geometry, Lie groups, representation theory, signal processing, quantum physics, medicine, and engineering, just to name few influential connections. In particular, considerable progress has been made on Radon and wavelet transforms that is motivated by their wide range of applications. New questions in applications have inspired new questions and problems in mathematics.

Two main themes of the book are integral geometry and integral transforms, including the Radon transforms and the wavelet transform. Wavelets and related basis constructions, such as frames from engineering, are a little more than two decades old (not counting Haar's wavelet). Their theory and applications are presented from different angles: fractal analysis, harmonic analysis, operator theory, geometry, computations, special tilings, and algorithms. The use of wavelets is compared with other transform tools: Fourier, Radon, Gabor, multiscale and more.

Modern Integral geometry is the study of Radon transforms-integral transforms that are defined geometrically, as integrals over subsets. They are named after the Austrian mathematician, Johann Radon (1887-1956), who studied the transform that integrates planar functions over all lines in the plane. The goal is to learn about the function f from its integrals. This example is the mathematical model of X-ray tomography, and related transforms are appear in many other important tomography problems. Radon transforms are used in partial differential equations and group theory as well. The interplay between groups and Fourier and Radon transforms, including microlocal and harmonic analysis, provides the bridge between this theme and the other themes of this book.

The subjects covered in this book form a unified whole, and they stand at the crossroads of pure and applied mathematics. Together the topics of Radon transforms, geometry and wavelets represent a panoramic view of what is now called *Applied and Numerical Harmonic Analysis*. The interplay between the themes

#### PREFACE

of this volume is especially appealing. Wavelet theory can be used with Radon transforms to develop directional wavelets, and these can be used in microlocal analysis. Current researchers in Blashke-Gelfand-Santalo-style integral geometry use Fourier and microlocal analysis to understand classical problems. The interplay between Lie theory and Radon transforms is illustrated by several articles in this proceedings. Ideas from wavelet theory give rise to questions in the theory of group representation theory. As represented in this volume, problems on symmetric spaces and geometry can be understood and analyzed using integral geometry and integral transforms. The Fourier, wavelet and similar transforms discussed in this volume are used in PDEs. Some new trends include the use of geometries and diagrams in the extension of the concept of wavelet sets, e.g., Bratteli diagrams, Coxeter, and Dynkin diagrams!

This volume of Contemporary Mathematics is based on two special sessions at the annual AMS meeting in New Orleans, January 2007 and a satellite workshop in Baton Rouge, January 4-5, 2007, on Harmonic Analysis and Applications. The sections are: Special Session on Radon Transforms, Convex Geometry, and Geometric Analysis organized by Eric L. Grinberg, Peter Kuchment, Gestur Ólafsson, Eric Todd Quinto, and Boris S. Rubin; Special Session on Frames and Wavelets in Harmonic Analysis, Geometry, and Applications organized by Palle E. T. Jorgensen, David R. Larson, Peter R. Massopust, and Gestur Ólafsson. The workshop at LSU was organized by Gestur Ólafsson and Boris Rubin and was supported by NSF grant DMS-0637383. All articles in this volume have been peer-reviewed.

This volume consists of invited expositions which together represent a broad spectrum of fields, stressing surprising interactions and connections between areas that are normally thought of as disparate. On the relatively pure side are harmonic analysis, convex geometry, symmetric spaces, representation theory (the groups include continuous and discrete, finite and infinite, compact and non-compact), operator theory, PDE, and mathematical probability. Moving in the applied direction are wavelets, fractals, ergodic theory, engineering topics such as frames, signal and image processing, including medical imaging, and mathematical physics. Work on Radon transforms and their applications spans almost a century, and it continues to inspire students and to interact with both pure and applied mathematics (e.g., tomography, PDE, and Gabor time-frequency analysis). Although the articles cover a broad range in harmonic analysis, the main themes are related to integral geometry, the Radon transform, wavelets and frame theory. We group them in the following way, but any such classification reflects only some aspects of the individual articles.

#### Frame Theory and applications

- (1) J.J. Benedetto, O. Oktay, A. Tangboondouangjit: Complex Sigma-Delta Quantization Algorithms for Finite Frames.
- (2) B. Johnson and K. Okoudjou: Frame Potential and Finite Abelian Groups.
- (3) G. Kutyniok and Casazza: Robustness of Fusion Frames under Erasures of Subspaces and of Local Frame Vectors.

#### Harmonic Analysis and Function Spaces

- (4) G. Ólafsson and S. Zheng: Harmonic Analysis Related to Schrödinger Operators.
- (5) I. Pesenson: A Discrete Helgason-Fourier Transform for Sobolev and Besov Functions on Noncompact Symmetric Spaces.

#### Harmonic Analysis and Number Theory

(6) D. Hart and A. Iosevich: Sums and Products in Finite Fields: An Integral Geometric Viewpoint.

#### **Integral Geometry and Radon Transforms**

- (7) D. Feldman: A Computational Complexity Paradigm for Tomography.
- (8) F. Gonzalez: Invariant Differential Operators on Matrix Motion Groups and Applications to the Matrix Radon Transform.
- (9) G. Olafsson and B. Rubin: Invariant Functions on Grassmanians.
- (10) E.T. Quinto: Helgason's Support Theorem and Spherical Radon Transforms.

#### Multiresolution Analysis, Wavelets, and Applications

- (11) B. Currey and T. McNamara: Decomposition and Admissibility for the Quasiregular Representation for Generalized Oscillator Groups.
- (12) D. Dutkay and P. Jorgensen: Fourier Series on Fractals: A Parallel with Wavelet Theory.
- (13) I. Aliev, B. Rubin, S. Sezer and S. Uyhan Composite Wavelet Transforms: Applications and Perspectives.
- (14) S. Jain, M. Papadakis and E. Dussaud: Explicit Schemes in Seismic Migration and Isotropic Multiscale Representations.
- (15) K. Merrill: Smooth, Well-localized Parseval Wavelets Based on Wavelet Sets in  $\mathbb{R}^2$ .

We would like to thank Louisiana State University and the National Science Foundation for their support. We thank the American Mathematical Society for its help organizing the special sessions that motivated this volume. We thank Christine Thivierge and the AMS Publications Department for its cooperation with the editors and their able and thorough job producing these proceedings. We thank the participants in the conferences for the presentation of their work, stimulating discussion, and for their good company. We thank the authors for submitting highquality articles. We hope you find this volume helpful, stimulating, and enjoyable.

The editors: Eric Grinberg, Palle Jorgensen, David Larson, Peter Massopust, Todd Quinto, Boris Rubin, and head editor Gestur Ólafsson.

#### Combined list of speakers and titles of their talks

Note that some speakers gave talks in the conference at LSU in Baton Rouge as well as the AMS national meeting in New Orleans. G. Ambartsoumian, The University of Texas at Arlington had planned to give a talk on *On reconstruction in limited view tomography*, but was not able to come. Instead B. Rubin, LSU, gave a talk.

- (1) A. Beltukov, University of the Pacific: Operator Identities Relating Sonar and Radon Transforms.
- (2) J. J. Benedetto, Norbert Wiener Center, University of Maryland, College Park: Pointwise Comparison of Pulse Code and Sigma-Delta Modulation.
- (3) P. G. Casazza, University of Missouri: The Kadison-Singer Problem in Frame Theory and Harmonic Analysis.
- (4) J. G. Christensen, Louisiana State University: Gelfand Triples and Time Frequency Analysis.
- (5) B. Currey, St. Louis University: Admissibility for Quasiregular Representations of Algebraic Solvable Lie Groups.
- (6) M. Dobrescu, Christopher Newport University: Coxeter Groups and Wavelets.
- (7) Leon Ehrenpreis, Temple University: Parametric and Nonparametric Radon Transforms.
- (8) M. Fickus, Air Force Institute of Technology: Maximally Equiangular Frames and Finite Wigner Distributions.
- (9) D. V. Finch: Oregon State University Integral Geometric Problems Arising in Thermoacoustic Tomography.
- (10) V. Furst, University of Arizona: A Characteristic Equation of Semiorthogonal Parseval Wavelets.
- (11) F. B. Gonzalez, Tufts University: The Modified Wave Equation on the Sphere and Extensions to Compact Symmetric Spaces.
- (12) A. Greenleaf, University of Rochester: Microlocal Analysis of the Linearized Attenuated Radon transform.
- (13) E. Grinberg, University of New Hampshire: The Gauss-Bonnet-Grotemeyer Theorem in Spaces of Constant Curvature.
- (14) D. Hardin, Vanderbilt University: Orthogonal Wavelets Centered on an Arbitrary Knot Sequence.
- (15) S. Helgason, MIT: An Inversion Formula for the X-ray Transform on a Compact Symmetric Space.
- (16) T. Henderson, United States Military Academy: Causal Relationships Between Frames.
- (17) K. R. Hoover, University of Oregon:
  - Dimension Functions of Rationally Dilated Wavelets

#### COMBINED LIST OF SPEAKERS AND TITLES OF THEIR TALKS

- The Dimension Function Of A Rationally Dilated Wavelet Associated With A GMRA.
- (18) R. Howard: University of South Carolina:
  - Results and Conjectures on the "Size" of the Space of Smooth Functions on a Compact Manifold
  - An Injectivity Theorem for Radon Transforms Restricted to Isotropic Functions.
- (19) A. Iosevich, University of Missouri-Columbia: Incidence Theory from Discrete and Analytical Perspectives.
- (20) B. Johnson, St. Louis University: Frame Decomposition of Principal Shiftinvariant Spaces with Rational Dilations.
- (21) P. E. T. Jorgensen, University of Iowa:
  - $C^*$ -algebras, Fractals, Wavelets and Dynamics
  - Bases and Frames in L<sup>2</sup>-spaces in Affine Iterated Function Systems (IFS).
- (22) T. Kakehi, University of Tsukuba, Japan: The Invariant Differential Operators on Cartan Motion Groups and Range Characterizations for Radon Transforms.
- (23) M. Kapralov, University of Central Florida: A 1PI Algorithm for Helical Trajectories That Violate the Convexity Condition.
- (24) B. Kjos-Hanssen, University of Connecticut: Some Computably Random Series of Functions.
- (25) A. Koldobsky, University of Missouri-Columbia:
  - Intersection Bodies and L<sub>p</sub>-spaces
  - Determination of Convex Bodies from Derivatives of Section Functions.
- (26) K. Kornelson, Grinnell College: IFSs with Overlap: Families of Orthogonal Exponentials and Invariant Measures, Part 1.
- (27) D. Larson/G. Olafsson, Texas A&M/LSU: Triple Wavelet Sets.
- (28) M. Ludwig, Technische Universität Wien:  $L_p$  Intersection Bodies.
- (29) T. McNamara, St. Louis University: Explicit Construction of Representations for a Class of Nilpotent Lie Groups and Their Application to Continuous Wavelet Transforms.
- (30) K. D. Merrill, Colorado College: Smooth, Well-localized Frame Wavelets Based on New Simple Wavelet Sets in R<sup>2</sup>.
- (31) N. Q. Nguyen, Texas A&M University:
  - Surgery on Frames
  - Surgery and Push-outs on Frames.
- (32) M. Nielsen: On Quasi-greedy Uniformly Bounded Bases for  $L_p([0,1])$ .
- (33) K. A. Okoudjou, University of Maryland: Uncertainty Principle for Fractals, Graphs and Metric Measure Spaces.
- (34) E. Ournycheva, Kent State University:
  - Two Spaces Conditions for Integrability of the Fourier Transform
  - Semyanistyi's Integrals and Radon Transforms on Matrix Spaces.
- (35) J. A. Packer, University of Colorado, Boulder: Isometries Arising from Filter Functions and Wavelets.
- (36) J-C. A. Paiva, Universite des Sciences et Technologies de Lille Some Applications of Integral Geometry to Finsler Geometry.

- (37) M. Papadakis, University of Houston: Isotropic Multiresolution Analysis.
- (38) I. Pesenson, Temple University: Bernstein-Nikolskii Inequality on Symmetric Spaces.
- (39) E. T. Quinto, Tufts:
  - Support Theorems for the Spherical Radon Transform on Manifolds
  - Local Tomography in SPECT.
- (40) B. Rubin, LSU: The Busemann-Petty Problem with the Generalized Axial Symmetry.
- (41) D. Ryabogin, Kansas State University: On the Local Equatorial Characterization of Zonoids.
- (42) K. L. Shuman, Grinnell College: IFSs with Overlap: Families of Orthogonal Exponentials and Invariant Measures, Part 2.
- (43) L. Slavin, University of Connecticut: Bellman Function and a Local Chang-Wilson-Wolff Theorem.
- (44) M-S. Song: Entropy Encoding in Wavelet Image Compression.
- (45) D. Speegle, St. Louis University: The Beurling Dimension of Gabour Pseudoframes of Affine Subspaces.
- (46) Z. I. Szabo, Lehman College and Graduate Center of the City University of New York
  - Z. I. Szabo, CUNY: De Broglie Geometry on Zeeman Manifolds: A New Non-perturbative Approach to the Infinities of QED (Review of the Scalar Theory)
  - Isospectral Metrics on Balls, Spheres, and Other Manifolds with Different Local Geometries.
- (47) M. Yaskina, University of Oklahoma:
  - Non-intersection Bodies All of Whose Central Sections are Intersection Bodies
  - The Geometry of  $L_0$ .
- (48) V. Yaskin, University of Oklahoma:
  - Inequalities of the Kahane-Khinchin Type and Sections of  $L_p$ -balls
  - A Solution to the Lower Dimensional Busemann-Petty Problem in the Hyperbolic Space.
- (49) A. I. Zayed, DePaul University: Texture Identification of Tissues Using Directional Wavelet, Ridgelet and Curvelet Transforms.
- (50) S. Zheng, University of South Carolina: Spectral Calculus in Function Spaces and Dispersive Equations.
- (51) Y. Wang, Georgia Institute of Technology/NSF: Wavelet Sets with Nonexpanding Dilation Matrices.
- (52) A. Zvavitch, Kent State University: There is No Local Characterization of Intersection Bodies.

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The subjects covered in this book form a unified whole, and they stand at the crossroads of pure and applied mathematics. The articles cover a broad range in harmonic analysis, with the main themes related to integral geometry, the Radon transform, wavelets and frame theory. These themes can loosely be grouped together as follows:

- Frame Theory and Applications
- · Harmonic Analysis and Function Spaces
- Harmonic Analysis and Number Theory
- Integral Geometry and Radon Transforms
- Multiresolution Analysis, Wavelets, and Applications



