

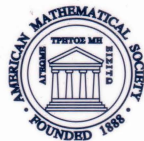
CONTEMPORARY MATHEMATICS

473

Stationary and Time Dependent Gross-Pitaevskii Equations

Wolfgang Pauli Institute 2006 Thematic Program
January–December, 2006
Vienna, Austria

Alberto Farina
Jean-Claude Saut
Editors



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Preface

The Gross-Pitaevskii equation (an example of a defocusing nonlinear Schrödinger equation) describes phenomena such as the Bose-Einstein condensation of ultra cold atomic gases, the superfluidity of Helium II, or the "dark solitons" of Nonlinear Optics. Actually, many interesting and difficult mathematical questions associated with the Gross-Pitaevskii equation (linked for instance to the nontrivial boundary conditions at infinity) arise naturally from its modelling aspects. In particular, the long time dynamics of the Gross-Pitaevskii equation is highly nontrivial despite it being a defocusing nonlinear Schrödinger equation...

These Proceedings are issued from a Thematic Program ("Stationary and Time Dependent Gross-Pitaevskii equations") organized by Alberto Farina, Peter Markowitch and Jean-Claude Saut at the Wolfgang Pauli Institute (WPI) in Vienna during the year 2006. Two Workshops and a Summer School were organized, gathering about forty mathematicians and physicists. The contributions therein review some of the recent developments in the mathematical theory of the Gross-Pitaevskii equation. In particular, the following aspects are considered: modelling of superfluidity and Bose-Einstein condensation; Cauchy problem; the semi-classical limit; scattering theory; existence and properties of coherent travelling structures.

The Thematic Program was made possible thanks to the generous hospitality and financial support of the Wolfgang Pauli Institute. The excellent working conditions there have facilitated the interactions between all participants and we hope that these Proceedings will reflect some of those interactions.

We are especially thankful to all the participants to the Thematic Program, to the WPI Director, Professor Norbert Mauser and to its technical staff, in particular Mrs. Stefanie Preuss.

Alberto Farina, Jean-Claude Saut

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This volume is based on a thematic program on the Gross–Pitaevskii equation, which was held at the Wolfgang Pauli Institute in Vienna in 2006. The program consisted of two workshops and a one-week Summer School.

The Gross–Pitaevskii equation, an example of a defocusing nonlinear Schrödinger equation, is a model for phenomena such as the Bose–Einstein condensation of ultra cold atomic gases, the superfluidity of Helium II, or the “dark solitons” of Nonlinear Optics. Many interesting and difficult mathematical questions associated with the Gross–Pitaevskii equation, linked for instance to the nontrivial boundary conditions at infinity, arise naturally from its modeling aspects.

The articles in this volume review some of the recent developments in the theory of the Gross–Pitaevskii equation. In particular the following aspects are considered: modeling of superfluidity and Bose–Einstein condensation, the Cauchy problem, the semi-classical limit, scattering theory, existence and properties of coherent traveling structures, and numerical simulations.

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