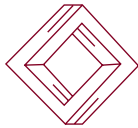


# CONTEMPORARY MATHEMATICS

476



**APORTACIONES MATEMÁTICAS**  
SOCIEDAD MATEMÁTICA MEXICANA

## Fourth Summer School in Analysis and Mathematical Physics

Topics in Spectral Theory  
and Quantum Mechanics

May 2005

Universidad Nacional Autónoma de México  
Cuernavaca, México

Carlos Villegas-Blas  
Editor



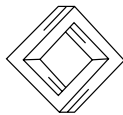
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## Introduction

Quantum mechanics started at the beginning of the last century as a revolutionary theory to explain certain experimental facts which were in contradiction with the classical physics theory. One of the very basic aspects of this new theory was to recognize the wave-like behavior of particles. As a consequence, the study of characteristic frequencies of a given system plays a key role in the theory. In mathematical terms, the spectral theory of Schrödinger operators turned out to be part of the base of Quantum Mechanics.

This volume is about three topics strongly related to the development of spectral theory. It consists of three expository articles written by outstanding researchers in mathematical physics and spectral theory: Rafael D. Benguria, Peter D. Hislop, Elliott H. Lieb and Helmut Linde. The articles are based on the lectures delivered by Benguria, Hislop and Lieb in the IV Summer School in analysis and mathematical physics that took place at the Institute of Mathematics, Universidad Nacional Autónoma de México, Cuernavaca, May 2005. The main goal of both the lectures and the articles is to link the basic knowledge of a graduate student in mathematics with three current research topics in mathematical physics: Isoperimetric inequalities for eigenvalues of the Laplace Operator, Random Schrödinger Operators and Stability of Matter. The above-mentioned authors have made decisive contributions to their corresponding research fields. Thus graduate students will find very well written articles guiding and introducing them into current research topics and specialist researchers will find information on recent progress in some areas of mathematical physics written by some of the people who have made it.

Finally let me mention that the article written by Elliott H. Lieb has already appeared in a previous publication: *The Stability of Matter and Quantum Electrodynamics*, the Jahresbericht of the German Math. Soc. JB **106**, 93-110 (2004) (Teubner). Professor Lieb based his lectures on this article and has been so kind to let us reproduce it in this volume in order to make it more complete as the proceedings of the above-mentioned summer school.

Carlos Villegas Blas, editor.  
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