

# CONTEMPORARY MATHEMATICS

506

## Quantum Affine Algebras, Extended Affine Lie Algebras, and Their Applications

Quantum Affine Algebras,  
Extended Affine Lie Algebras, and Applications  
March 2–7, 2008  
Banff International Research Station  
Banff, Canada

Yun Gao  
Naihuan Jing  
Michael Lau  
Kailash C. Misra  
Editors



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**American Mathematical Society**  
Providence, Rhode Island

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2000 *Mathematics Subject Classification*. Primary 17B10, 17B37, 17B65, 17B67.

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### Library of Congress Cataloging-in-Publication Data

Workshop on Quantum Affine Lie Algebras, Extended Affine Lie algebras, and Applications (2008 : Banff International Research Station)

Quantum affine algebras, extended affine Lie algebras, and their applications : Workshop on Quantum Affine Lie Algebras, Extended Affine Lie algebras, and Applications, March 2–7, 2008, Banff International Research Station, Banff, Canada / Yun Gao . . . [et al.], editors.

p. cm. — (Contemporary mathematics ; v. 506)

Includes bibliographical references.

ISBN 978-0-8218-4507-3 (alk. paper)

1. Geometry, Algebraic—Congresses. 2. Geometry, Affine—Congresses. I. Gao, Yun, 1963–  
II. Title.

QA564.W67 2008  
516'.4—dc22

2009037983

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10 9 8 7 6 5 4 3 2 1    15 14 13 12 11 10

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## Preface

Affine Lie algebras appear in many contexts of mathematics and mathematical physics. They have a Kac-Moody presentation in terms of the Serre relations, as well as a loop presentation in terms of maps from the punctured plane to a finite-dimensional simple Lie algebra. Both of these points of view have proved enormously profitable since their introduction 40 years ago, with applications to numerous areas, including Lie theory, number theory, modular forms, Hopf algebras, algebraic groups, combinatorics, knot invariants, quantum field theory, integrable systems, and statistical mechanics.

More recent work has concentrated on generalizations of affine Lie algebras, especially those involving  $q$ -deformations of affine structures and multivariable analogues of affine Lie algebras. Two of the most important such generalizations are quantum affine algebras and extended affine Lie algebras.

Quantum affine algebras are defined via a  $q$ -deformation of the Serre relations in the Kac-Moody presentation of affine Lie algebras. They combine the power of the  $\mathcal{R}$ -matrix of the braided tensor category of representations of a (finite) quantum group with a quantum version of the Knizhnik-Zamolodchikov connection on the bundle of conformal blocks in 2-dimensional WZW conformal field theory.

Extended affine Lie algebras are a multivariable generalization of the loop presentation of affine Lie algebras. They have appeared in the context of toroidal symmetries of higher dimensional quantum field theories, as well as in the solution of integrable hierarchies of soliton equations. Recent work by Allison, Berman, Faulkner, Neher, and Pianzola has given a new perspective on the structure theory of extended affine Lie algebras. The representation theory of extended affine Lie algebras is not yet well understood, but it is expected that a better understanding of these algebras will lead to many applications, as has already been the case in affine and quantum affine theory.

In March 2008, thirty-eight of the leading experts in these areas gathered at the Banff International Research Station in western Canada. There were 19 invited talks, as well as informal presentations and collaborations in smaller groups in the evenings. This volume contains 12 papers from that conference, and it illustrates some of the breadth and vitality of this rapidly expanding field. We thank Christine Thivierge at the AMS editorial office for her assistance in the preparation of this volume and the Banff International Research Station for its superb management of the conference logistics. Most of all, we thank the authors, participants, and anonymous referees who made the BIRS conference and this volume entirely possible.

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This volume contains the proceedings of the conference on Quantum Affine Algebras, Extended Affine Lie Algebras, and Applications, which was held at the Banff International Research Station, Banff, Canada, from March 2–7, 2008.

Many of the papers include new results on different aspects of quantum affine algebras, extended affine Lie algebras, and their applications in other areas of mathematics and physics. Any reader interested in learning about the recent developments in quantum affine algebras and extended affine Lie algebras will benefit from this book.

ISBN 978-0-8218-4507-3



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