Entropy and the Quantum

Arizona School of Analysis with Applications
March 16–20, 2009
University of Arizona

Robert Sims
Daniel Ueltschi

Editors

Entropy and the Quantum

CONTEMPORARY MATHEMATICS

Entropy and the Quantum

Arizona School of Analysis with Applications
March 16–20, 2009
University of Arizona

Robert Sims
Daniel Ueltschi
Editors

Contents

Preface

Interactions between mathematics and physics have often been complicated and, interestingly enough, quite fruitful. The main goal of the school *Entropy and the Quantum* was to give young mathematicians access to some of the intuitions of quantum mechanics and bring clarity to certain mathematics that has been motivated by this field. This too is the goal of the present lecture notes. They include an excellent introduction to quantum mechanics by William Faris, which explains many important ideas to a mathematical audience. Analytical inequalities have been developed over the years for, and because of, quantum mechanics. These are the subject of two contributions to this volume, one by Eric Carlen and one by Robert Seiringer. Far-fetched applications to the Shandrasekhar limit of stellar masses are explained by Christian Hainzl. Another topic related to quantum mechanics, and the object of much current research, is quantum information theory. The lecture notes of Bruno Nachtergaele and Robert Sims, and of Christopher King, are devoted to recent developments herein that are especially relevant to mathematics. These lecture notes have been written primarily for PhD students and for postdocs, but they will certainly be valuable to more experienced researchers as well.

These notes are motivated by the school *Entropy and the Quantum* that took place in Tucson, Arizona, on March 16-20, 2009. The participants were treated to beautiful lectures by Carlen, Nachtergaele, Seiringer, and Wehr. There were also regular talks by Betz, Hainzl, Kawai, Newman, Rey-Bellet, and Ruskai. This event was a pleasant week on the sunny campus of the University of Arizona, with invigorating, post-lecture discussions at a variety of places near the campus. The excursion to the Arizona-Sonora Desert Museum, near Tucson, constituted a welcome break and a memorable introduction to the wildlife of the area.

The organizers would like to thank the many people who made this pleasant and useful school possible. Tom Kennedy and Jan Wehr gave encouragements and advice, Bill Faris suggested the catchy (and accurate!) title of the school. The enthusiasm and dedication of the lecturers were vital. The efficiency and patience of Annette Horn, the conference coordinator, allowed the event to run smoothly. We also want to thank our sponsors: the University of the Arizona, for financial support and for use of their facilities; and the National Science Foundation, which provided the bulk of the financial support (grant DMS-0852422). Finally, the organizers thank all the participants of *Entropy and the Quantum* for their interest, for their questions, and for their good humor.

Tucson, Arizona, April 7th, 2010
Robert Sims, Daniel Ueltschi

List of Participants

Bachmann Sven
ETH Zürich

Betz Volker
University of Warwick

Bishop Michael
University of Arizona

Borovyk Vita
University of Arizona

Bouch Gabe
Rutgers University

Carlen Eric
Rutgers University

Chen Y.C.
University of Arizona

Dahl Janine
Rice University

De Oliveira Gustavo
University of British Columbia

Demirel Semra
University of Stuttgart

Djordjevic Ivan
University of Arizona

Faris Bill
University of Arizona

Fatkulin Ibrahim
University of Arizona

Frank Rupert
Princeton University

Freiji Abraham
University of Alabama at Birmingham

Gan Zheng
Rice University

Greenblatt Rafael
Rutgers University

Guevara Cristi
Arizona State University

Hainzl Christian
University of Alabama, Birmingham

Halasan Florina
University of British Columbia

Hamza Eman
Michigan State University

Hermi Lotfi
University of Arizona

Kawai Ryoichi
University of Alabama at Birmingham

Kennedy Tom
University of Arizona

Kerl John
University of Arizona

King Christopher
Northeastern University

Knowles Antti
ETH Zürich

Koldan Nilufer
Northeastern University

LaGatta Tom
University of Arizona

Lin Kevin
University of Arizona

Michelangeli Alessandro
SISSA Trieste

Morales David
University of Arizona

Moser David
Northeastern University

Nachtergaele Bruno
UC Davis

Nathanson Michael
St. Mary's College

Newman Charles
New York University

Ng Stephen
UC Davis

Nichols Roger
University of Alabama at Birmingham

Pickrell Doug
University of Arizona

Pogorzelska Monika
Uniwersytetu Gdańskiego

Raz Hillel
UC Davis

Rey-Bellet Luc
University of Massachusetts

Ruskai Mary Beth
Tufts University

Seiringer Robert
Princeton University

Shulei
University of Arizona

Sims Robert
University of Arizona

Terra Cunha Marcelo
Universidade Federal de Minas Gerais

Tyson Jon
Harvard University

Ueltschi Daniel
University of Warwick

Wehr Jan
University of Arizona

Wouters Jeroen
University of Leuven

Yildirim-Yolcu Selma
Georgia Institute of Technology

Young Brent
Rutgers University

Titles in This Series

TITLES IN THIS SERIES

For a complete list of titles in this series, visit the
AMS Bookstore at **www.ams.org/bookstore/**.

These lecture notes provide a pedagogical introduction to quantum mechanics and to some of the mathematics that has been motivated by this field. They are a product of the school "Entropy and the Quantum", which took place in Tucson, Arizona, in 2009. They have been written primarily for young mathematicians, but they will also prove useful to more experienced analysts and mathematical physicists. In the first contribution, William Faris introduces the mathematics of quantum mechanics. Robert Seiringer and Eric Carlen review certain recent developments in stability of matter and analytic inequalities, respectively. Bruno Nachtergaele and Robert Sims review locality results for quantum systems, and Christopher King deals with additivity conjectures and quantum information theory. The final article, by Christian Hainzl, describes applications of analysis to the Shandrasekhar limit of stellar masses.