

CONTEMPORARY MATHEMATICS

529

Entropy and the Quantum

Arizona School of Analysis with Applications
March 16–20, 2009
University of Arizona

Robert Sims
Daniel Ueltschi
Editors



American Mathematical Society

Entropy and the Quantum

CONTEMPORARY MATHEMATICS

529

Entropy and the Quantum

Arizona School of Analysis with Applications
March 16–20, 2009
University of Arizona

Robert Sims
Daniel Ueltschi
Editors



American Mathematical Society
Providence, Rhode Island

Editorial Board

Dennis DeTurck, managing editor

George Andrews Abel Klein Martin J. Strauss

2000 *Mathematics Subject Classification*. Primary 15A90, 47A63, 81P45, 81Q10, 81Q15, 81V17, 82C10, 82C20, 94A40.

Library of Congress Cataloging-in-Publication Data

Arizona School of Analysis with Applications (2009 : University of Arizona)

Entropy and the quantum : Arizona School of Analysis with Applications, March 16–20, 2009, University of Arizona / Robert Sims, Daniel Ueltschi, editors.

p. cm. — (Contemporary mathematics ; v. 529)

Includes bibliographical references.

ISBN 978-0-8218-5247-7 (alk. paper)

1. Quantum Entropy—Congresses. I. Sims, Robert, 1975– II. Ueltschi, Daniel, 1969– III. Title.

QC174.85.Q83A75 2010

530.12—dc22

2010024656

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2010 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Copyright of individual articles may revert to the public domain 28 years
after publication. Contact the AMS for copyright status of individual articles.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 15 14 13 12 11 10

Contents

Preface	vii
List of Participants	ix
Outline of Quantum Mechanics WILLIAM G. FARIS	1
Inequalities for Schrödinger Operators and Applications to the Stability of Matter Problem ROBERT SEIRINGER	53
Trace Inequalities and Quantum Entropy: An Introductory Course ERIC CARLEN	73
Lieb-Robinson Bounds in Quantum Many-Body Physics BRUNO NACHTERGAELE and ROBERT SIMS	141
Remarks on the Additivity Conjectures for Quantum Channels CHRISTOPHER KING	177
On the Static and Dynamical Collapse of White Dwarfs CHRISTIAN HAINZL	189

Preface

Interactions between mathematics and physics have often been complicated and, interestingly enough, quite fruitful. The main goal of the school *Entropy and the Quantum* was to give young mathematicians access to some of the intuitions of quantum mechanics and bring clarity to certain mathematics that has been motivated by this field. This too is the goal of the present lecture notes. They include an excellent introduction to quantum mechanics by William Faris, which explains many important ideas to a mathematical audience. Analytical inequalities have been developed over the years for, and because of, quantum mechanics. These are the subject of two contributions to this volume, one by Eric Carlen and one by Robert Seiringer. Far-fetched applications to the Shandrasekhar limit of stellar masses are explained by Christian Hainzl. Another topic related to quantum mechanics, and the object of much current research, is quantum information theory. The lecture notes of Bruno Nachtergaele and Robert Sims, and of Christopher King, are devoted to recent developments herein that are especially relevant to mathematics. These lecture notes have been written primarily for PhD students and for postdocs, but they will certainly be valuable to more experienced researchers as well.

These notes are motivated by the school *Entropy and the Quantum* that took place in Tucson, Arizona, on March 16-20, 2009. The participants were treated to beautiful lectures by Carlen, Nachtergaele, Seiringer, and Wehr. There were also regular talks by Betz, Hainzl, Kawai, Newman, Rey-Bellet, and Ruskai. This event was a pleasant week on the sunny campus of the University of Arizona, with invigorating, post-lecture discussions at a variety of places near the campus. The excursion to the Arizona-Sonora Desert Museum, near Tucson, constituted a welcome break and a memorable introduction to the wildlife of the area.

The organizers would like to thank the many people who made this pleasant and useful school possible. Tom Kennedy and Jan Wehr gave encouragements and advice, Bill Faris suggested the catchy (and accurate!) title of the school. The enthusiasm and dedication of the lecturers were vital. The efficiency and patience of Annette Horn, the conference coordinator, allowed the event to run smoothly. We also want to thank our sponsors: the University of the Arizona, for financial support and for use of their facilities; and the National Science Foundation, which provided the bulk of the financial support (grant DMS-0852422). Finally, the organizers thank all the participants of *Entropy and the Quantum* for their interest, for their questions, and for their good humor.

Tucson, Arizona, April 7th, 2010
Robert Sims, Daniel Ueltschi

List of Participants

Bachmann Sven
ETH Zürich

Betz Volker
University of Warwick

Bishop Michael
University of Arizona

Borovyk Vita
University of Arizona

Bouch Gabe
Rutgers University

Carlen Eric
Rutgers University

Chen Y.C.
University of Arizona

Dahl Janine
Rice University

De Oliveira Gustavo
University of British Columbia

Demirel Semra
University of Stuttgart

Djordjevic Ivan
University of Arizona

Faris Bill
University of Arizona

Fatkulin Ibrahim
University of Arizona

Frank Rupert
Princeton University

Freiji Abraham
University of Alabama at Birmingham

Gan Zheng
Rice University

Greenblatt Rafael
Rutgers University

Guevara Cristi
Arizona State University

Hainzl Christian
University of Alabama, Birmingham

Halasan Florina
University of British Columbia

Hamza Eman
Michigan State University

Hermi Lotfi
University of Arizona

Kawai Ryoichi
University of Alabama at Birmingham

Kennedy Tom
University of Arizona

Kerl John
University of Arizona

King Christopher
Northeastern University

Knowles Antti
ETH Zürich

Koldan Nilufer
Northeastern University

LaGatta Tom
University of Arizona

Lin Kevin
University of Arizona

Michelangeli Alessandro
SISSA Trieste

Morales David
University of Arizona

Moser David
Northeastern University

Nachtergaele Bruno
UC Davis

Nathanson Michael
St. Mary's College

Newman Charles
New York University

Ng Stephen
UC Davis

Nichols Roger
University of Alabama at Birmingham

Pickrell Doug
University of Arizona

Pogorzelska Monika
Uniwersytetu Gdańskiego

Raz Hillel
UC Davis

Rey-Bellet Luc
University of Massachusetts

Ruskai Mary Beth
Tufts University

Seiringer Robert
Princeton University

Shulei
University of Arizona

Sims Robert
University of Arizona

Terra Cunha Marcelo
Universidade Federal de Minas Gerais

Tyson Jon
Harvard University

Ueltschi Daniel
University of Warwick

Wehr Jan
University of Arizona

Wouters Jeroen
University of Leuven

Yildirim-Yolcu Selma
Georgia Institute of Technology

Young Brent
Rutgers University

Titles in This Series

- 530 **Vitaly Bergelson, Andreas Blass, Mauro Di Nasso, and Renling Jin, Editors**, Ultrafilters across mathematics, 2010
- 529 **Robert Sims and Daniel Ueltschi, Editors**, Entropy and the quantum, 2010
- 528 **Alberto Farina and Enrico Valdinoci, Editors**, Symmetry for elliptic PDEs, 2010
- 527 **Ricardo Castaño-Bernard, Yan Soibelman, and Ilia Zharkov, Editors**, Mirror symmetry and tropical geometry, 2010
- 526 **Helge Holden and Kenneth H. Karlsen, Editors**, Nonlinear partial differential equations and hyperbolic wave phenomena, 2010
- 525 **Manuel D. Contreras and Santiago Díaz-Madriral, Editors**, Five lectures in complex analysis, 2010
- 524 **Mark L. Lewis, Gabriel Navarro, Donald S. Passman, and Thomas R. Wolf, Editors**, Character theory of finite groups, 2010
- 523 **Aiden A. Bruen and David L. Wehlau, Editors**, Error-correcting codes, finite geometries and cryptography, 2010
- 522 **Oscar García-Prada, Peter E. Newstead, Luis Álvarez-Cónsul, Indranil Biswas, Steven B. Bradlow, and Tomás L. Gómez, Editors**, Vector bundles and complex geometry, 2010
- 521 **David Kohel and Robert Rolland, Editors**, Arithmetic, geometry, cryptography and coding theory 2009, 2010
- 520 **Manuel E. Lladser, Robert S. Maier, Marni Mishna, and Andrew Rechnitzer, Editors**, Algorithmic probability and combinatorics, 2010
- 519 **Yves Félix, Gregory Lupton, and Samuel B. Smith, Editors**, Homotopy theory of function spaces and related topics, 2010
- 518 **Gary McGuire, Gary L. Mullen, Daniel Panario, and Igor E. Shparlinski, Editors**, Finite fields: Theory and applications, 2010
- 517 **Tewodros Amdeberhan, Luis A. Medina, and Victor H. Moll, Editors**, Gems in experimental mathematics, 2010
- 516 **Marlos A.G. Viana and Henry P. Wynn, Editors**, Algebraic methods in statistics and probability II, 2010
- 515 **Santiago Carrillo Menéndez and José Luis Fernández Pérez, Editors**, Mathematics in finance, 2010
- 514 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors**, Nonlinear analysis and optimization II, 2010
- 513 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors**, Nonlinear analysis and optimization I, 2010
- 512 **Albert Fathi, Yong-Geun Oh, and Claude Viterbo, Editors**, Symplectic topology and measure preserving dynamical systems, 2010
- 511 **Luise-Charlotte Kappe, Arturo Magidin, and Robert Fitzgerald Morse, Editors**, Computational group theory and the theory of groups, II, 2010
- 510 **Mario Bonk, Jane Gilman, Howard Masur, Yair Minsky, and Michael Wolf, Editors**, In the Tradition of Ahlfors-Bers, V, 2010
- 509 **Primitivo B. Acosta-Humánez and Francisco Marcellán, Editors**, Differential algebra, complex analysis and orthogonal polynomials, 2010
- 508 **Martin Berz and Khodr Shamseddine, Editors**, Advances in p -Adic and non-archimedean analysis, 2010
- 507 **Jorge Arvesú, Francisco Marcellán, and Andrei Martínez-Finkelshtein, Editors**, Recent trends in orthogonal polynomials and approximation theory, 2010
- 506 **Yun Gao, Naihuan Jing, Michael Lau, and Kailash C. Misra, Editors**, Quantum affine algebras, extended affine Lie algebras, and their applications, 2010
- 505 **Patricio Cifuentes, José García-Cuerva, Gustavo Garrigós, Eugenio Hernández, José María Martell, Javier Parcet, Alberto Ruiz, Fernando Soria, José Luis Torrea, and Ana Vargas, Editors**, Harmonic analysis and partial differential equations, 2010

TITLES IN THIS SERIES

- 504 **Christian Ausoni, Kathryn Hess, and Jérôme Scherer, Editors**, *Alpine perspectives on algebraic topology*, 2009
- 503 **Marcel de Jeu, Sergei Silvestrov, Christian Skau, and Jun Tomiyama, Editors**, *Operator structures and dynamical systems*, 2009
- 502 **Viviana Ene and Ezra Miller, Editors**, *Combinatorial Aspects of Commutative Algebra*, 2009
- 501 **Karel Dekimpe, Paul Igodt, and Alain Valette, Editors**, *Discrete groups and geometric structures*, 2009
- 500 **Philippe Briet, François Germinet, and Georgi Raikov, Editors**, *Spectral and scattering theory for quantum magnetic systems*, 2009
- 499 **Antonio Giambruno, César Polcino Milies, and Sudarshan K. Sehgal, Editors**, *Groups, rings and group rings*, 2009
- 498 **Nicolau C. Saldanha, Lawrence Conlon, Rémi Langevin, Takashi Tsuboi, and Pawel Walczak, Editors**, *Foliations, geometry and topology*, 2009
- 497 **Maarten Bergvelt, Gaywalee Yamskulna, and Wenhua Zhao, Editors**, *Vertex operator algebras and related areas*, 2009
- 496 **Daniel J. Bates, GianMario Besana, Sandra Di Rocco, and Charles W. Wampler, Editors**, *Interactions of classical and numerical algebraic geometry*, 2009
- 495 **G. L. Litvinov and S. N. Sergeev, Editors**, *Tropical and idempotent mathematics*, 2009
- 494 **Habib Ammari and Hyeonbae Kang, Editors**, *Imaging microstructures: Mathematical and computational challenges*, 2009
- 493 **Ricardo Baeza, Wai Kiu Chan, Detlev W. Hoffmann, and Rainer Schulze-Pillot, Editors**, *Quadratic Forms—Algebra, Arithmetic, and Geometry*, 2009
- 492 **Fernando Giráldez and Miguel A. Herrero, Editors**, *Mathematics, Developmental Biology and Tumour Growth*, 2009
- 491 **Carolyn S. Gordon, Juan Tirao, Jorge A. Vargas, and Joseph A. Wolf, Editors**, *New developments in Lie theory and geometry*, 2009
- 490 **Donald Babbitt, Vyjayanthi Chari, and Rita Fioresi, Editors**, *Symmetry in mathematics and physics*, 2009
- 489 **David Ginzburg, Erez Lapid, and David Soudry, Editors**, *Automorphic Forms and L -functions II. Local aspects*, 2009
- 488 **David Ginzburg, Erez Lapid, and David Soudry, Editors**, *Automorphic forms and L -functions I. Global aspects*, 2009
- 487 **Gilles Lachaud, Christophe Ritzenthaler, and Michael A. Tsfasman, Editors**, *Arithmetic, geometry, cryptography and coding theory*, 2009
- 486 **Frédéric Mynard and Elliott Pearl, Editors**, *Beyond topology*, 2009
- 485 **Idris Assani, Editor**, *Ergodic theory*, 2009
- 484 **Motoko Kotani, Hisashi Naito, and Tatsuya Tate, Editors**, *Spectral analysis in geometry and number theory*, 2009
- 483 **Vyacheslav Futorny, Victor Kac, Iryna Kashuba, and Efim Zelmanov, Editors**, *Algebras, representations and applications*, 2009
- 482 **Kazem Mahdavi and Deborah Koslover, Editors**, *Advances in quantum computation*, 2009
- 481 **Aydın Aytuna, Reinhold Meise, Tosun Terzioğlu, and Dietmar Vogt, Editors**, *Functional analysis and complex analysis*, 2009
- 480 **Nguyen Viet Dung, Franco Guerriero, Lakhdar Hammoudi, and Pramod Kanwar, Editors**, *Rings, modules and representations*, 2008
- 479 **Timothy Y. Chow and Daniel C. Isaksen, Editors**, *Communicating mathematics*, 2008

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/.

These lecture notes provide a pedagogical introduction to quantum mechanics and to some of the mathematics that has been motivated by this field. They are a product of the school “Entropy and the Quantum”, which took place in Tucson, Arizona, in 2009. They have been written primarily for young mathematicians, but they will also prove useful to more experienced analysts and mathematical physicists. In the first contribution, William Faris introduces the mathematics of quantum mechanics. Robert Seiringer and Eric Carlen review certain recent developments in stability of matter and analytic inequalities, respectively. Bruno Nachtergaele and Robert Sims review locality results for quantum systems, and Christopher King deals with additivity conjectures and quantum information theory. The final article, by Christian Hainzl, describes applications of analysis to the Shandrasekhar limit of stellar masses.

ISBN 978-0-8218-5247-7



9 780821 852477

CONM/529

AMS on the Web
www.ams.org