

# CONTEMPORARY MATHEMATICS

533

## Set Theory and Its Applications

Annual Boise Extravaganza  
in Set Theory  
Boise, Idaho  
1995–2010

L. Babinkostova  
A. E. Caicedo  
S. Geschke  
M. Scheepers  
Editors



American Mathematical Society

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Providence, Rhode Island

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## Preface

The Boise Extravaganza in Set Theory (BEST) started in 1992 as a small, locally funded conference dedicated to Set Theory and its Applications. A number of years after its inception BEST started being funded by the National Science Foundation. Without this funding it would not have been possible to maintain the conference.

The conference remained relatively small with many opportunities for its participants to meet informally. We like to think that during these years BEST has made it possible for the numerous set theorists who have participated in it to absorb, besides the new developments featured in the conference talks, also part of the folklore and traditions of the field of set theory and its relatives. An explicit effort was made to bring together role models from various career stages as well as the new generation to support some notion of continuity in the field. A list of these invited speakers follows this preface. The interested reader can find more information about the past BEST conferences at <http://math.boisestate.edu/~best>.

This volume has a similar purpose. In it the reader will find valuable papers ranging from surveys that make available knowledge that has been around for several decades as unpublished lore, to hybrid survey-research papers, to pure research papers. Readers can be assured of the authority of each paper since each has been carefully refereed. The reader will also find that the subjects treated in these papers range over several of the historically strongly represented areas of set theory and its relatives. Rather than expound the virtues of each paper individually here, we invite the reader to learn from the authors.

Bringing to publication such a collection of papers is not possible without the generous dedication of authors and referees and the services of a publisher. We would like to thank all authors and referees for their selfless contributions to this volume. And we particularly would like to thank the publisher, Contemporary Mathematics, and Christine Thivierge, for the guidance they provided during this process.

Liljana Babinkostova,  
Andrés E. Caicedo,  
Stefan Geschke  
and  
Marion Scheepers.

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