

CONTEMPORARY MATHEMATICS

551

Advances in non-Archimedean Analysis

11th International Conference

p -adic Functional Analysis

July 5–9, 2010

Université Blaise Pascal, Clermont-Ferrand, France

Jesus Araujo-Gomez

Bertin Diarra

Alain Escassut

Editors



Advances in non-Archimedean Analysis



Nicole De Grande-De Kimpe
(1936–2008)

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Photograph of Nicole De Grande-De Kimpe courtesy of Cristina Perez-Garcia.

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on p -adic Functional Analysis

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Preface

The eleventh International Conference on p -adic Functional Analysis was held at Université Blaise Pascal of Clermont-Ferrand, France, July 5-9, 2010. The first conference of this series was organized in 1990 by University of Cantabria in Laredo, Spain and the series continued with meetings every two years in various places. Particularly, the 10th conference was held at Michigan State University (East Lansing), the 9th was held at Concepcion, Chile and the 8th was already organized at Université Blaise Pascal. The previous ones were held at Nijmegen (the Netherland), Ioaninna (Greece)...

Now, nearly 40 researchers from 15 different countries attended the July conference. This book contains research articles based on some of the 30-minute talks given at the conference and present a wide range of recent advances in p -adic analysis.

Basic functional analysis is represented by contributions covering a wide range of topics: Hilbert and Banach spaces, finite dimensional spaces, locally convex spaces, Fréchet spaces, inner products, orthogonality, strict topologies, spaces of continuous functions and of strictly differentiable functions, operator theory: isomorphisms between Banach functions spaces, measure and integration: vector measures, dual space for strict topology, extrem and mean value in Levi-Civita fields, Banach and topological algebras, maximal ideals of the ultrametric corona algebra.

Functions from a p -adic field to \mathbb{C} , are considered: p -adic fractal strings, geometric zeta functions and tube formulas, elliptic pseudo-differential equations, Sobolev spaces.

Analytic and meromorphic functions remain a wide domain of research: differential equations and radius of convergence of solutions, Haar's measure and p -adic dynamical systems, q -difference equations, Lipschitz condition for rational functions, identities and congruences for Genocchi numbers.

In addition, the reader will find studies in the case where the ground field is a Krull valued field: functional analysis, orthomodular spaces, spherical completion of normed spaces, Lipschitz operators.

Participants to the previous conference in July 2008 and all specialists of p -adic analysis were very saddened to know that Nicole de Grande de Kimpe passed away. Her work in p -adic analysis and in other domains of analysis as well as in the organization of our meetings in the nineties were outstanding. This is why it was decided to dedicate this conference to her memory and the conference was opened with a presentation of her life and achievement.

We are grateful to Jose Aguayo Garrido, Gilles Christol, Herminia Ochsenius, Wim Schikhof, Athanasio Katsaras, Cristina Perez Garcia, Khodr Shamseddine, who, with the undersigned, made the Scientific Committee. And we thank the

Laboratoire de Mathématiques, UMR 6620, CNRS-UBP; Université Blaise Pascal, Conseil Régional de la Région Auvergne, Conseil Général du Puy de Dôme, and the township of Cermont-Ferrand for their financial support. We also wish to thank the American Mathematical Society for publishing this volume.

We are now looking forward to the next meeting in the series, which will be organized by Khodr Shamseddine at University of Manitoba, Winnipeg, Canada.

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The articles collected here feature recent developments in various areas of non-Archimedean analysis: Hilbert and Banach spaces, finite dimensional spaces, topological vector spaces and operator theory, strict topologies, spaces of continuous functions and of strictly differentiable functions, isomorphisms between Banach functions spaces, and measure and integration.

Other topics discussed in this volume include p -adic differential and q -difference equations, rational and non-Archimedean analytic functions, the spectrum of some algebras of analytic functions, and maximal ideals of the ultrametric corona algebra.

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