CONTEMPORARY MATHEMATICS

555

Commutative Algebra and Its Connections to Geometry

Pan-American Advanced Studies Institute
August 3–14, 2009
Universidade Federal de Pernambuco, Olinda, Brazil

Alberto Corso
Claudia Polini
Editors

American Mathematical Society

http://dx.doi.org/10.1090/conm/555

Commutative Algebra and Its Connections to Geometry

CONTEMPORARY MATHEMATICS

555

Commutative Algebra and Its Connections to Geometry

Pan-American Advanced Studies Institute
August 3–14, 2009
Universidade Federal de Pernambuco, Olinda, Brazil

Alberto Corso
Claudia Polini
Editors

American Mathematical Society
Providence, Rhode Island

2010 *Mathematics Subject Classification.* Primary 05C90, 13C40, 13D02, 13D07, 13D40, 14C05, 14J60, 14M12, 14N05.

Photographs courtesy of Alberto Corso.

Library of Congress Cataloging-in-Publication Data

Pan-American Advanced Studies Institute (2009 : Universidade Federal de Pernambuco)

Commutative algebra and its connections to geometry : Pan-american Advanced Studies Institute, August 3–14, 2009, Universidade Federal de Pernambuco, Olinda, Brazil / Alberto Corso, Claudia Polini, editors.

p. cm. — (Contemporary mathematics ; v. 555)

Includes bibliographical references.

ISBN 978-0-8218-4959-0 (alk. paper)

1. Combinatorial group theory—Congresses. 2. Algebra, Homological—Congresses. 3. Geometry, Algebraic—Congresses. I. Corso, Alberto. II. Polini, Claudia, 1966– III. Title.

QA182.5.P36 2009
512′.44—dc23

2011027283

Contents

Preface

Commutative algebra has several sources – among them number theory, invariant theory and algebraic geometry. Besides these classical sources, the field has undergone a striking evolution over the last quarter of a century, reaching out to applicable disciplines such as combinatorics, computer algebra, optimization and statistics. Commutative algebra is taught, and actively pursued as a research field, not only in developed countries, but also in several developing countries – in particular in Brazil and other Latin American countries such as Argentina and Mexico. In Brazil the area has since its inception been associated to algebraic geometry. This association has been very fruitful and, in particular, it has produced a joint research group (ALGA) involving partners in the scientific centers of the country.

Many of the results published in this volume were presented during the NSF-sponsored Pan-American Advanced Studies Institute (PASI) in *Commutative Algebra and its Connections to Geometry* held in Olinda (Brazil) during the period August 3-14, 2009, under the patronage of the Universidade Federal de Pernambuco. This institution is located in the center of the Brazilian Northeast, a vast, less developed region of the country. It has an established mathematics department with a long tradition as a research institution that has produced, for instance, a number of mathematicians currently holding positions at leading US universities.

The goals of this event were to reinforce the above mentioned successful Brazilian partnership throughout less privileged regions of Brazil and of Latin America in general by introducing young mathematicians from Brazil and other Latin American countries to fundamental techniques and recent developments in the field and promoting the collaboration between mathematicians from these areas and developed countries. It is our belief that the school contributed to the training of young researchers from North and South America in a timely atmosphere of international cooperation. For algebra in Latin America it had the effect of building regional expertise to overcome the problem of isolation. For both the US and the Latin American participants, the school and conference provided international experience and exposure, and new opportunities for collaborations. We expect that the meeting has in fact enlarged the circle of US researchers visiting Latin American universities on a regular basis. This in turn may help stem the now classical brain-drain of scientists from these countries. Yet another direction in which the school and conference succeeded was to enhance the collaboration between mathematicians within Latin America and to trigger initiatives for further interchange between the various countries of the continent.

We would also like to point out the fact that the PASI program was also an occasion to honor Wolmer Vasconcelos of Rutgers University, one of the most distinguished senior commutative algebraists. Wolmer Vasconcelos grew up in the

vicinity of Recife, where he also received his undergraduate education. He graduated with a Ph.D. in mathematics from the University of Chicago and spent most of his professional life at Rutgers University. So far he has supervised 17 doctoral students, authored 6 books, and written over 125 papers as a single author or jointly with a total of 40 collaborators. Wolmer Vasconcelos is a member of the Brazilian Academy of Sciences. Through his highly original contributions he has been a major influence in the development of modern commutative algebra.

The school and conference, co-organized by Craig Huneke, Aron Simis, Bernd Ulrich and the editors of the present volume, brought together more than 70 peridoctoral students from all over the world, but mostly from the Americas, and approximatively 50 senior researchers in the areas of commutative algebra and algebraic geometry. The focus of the program was on the following clusters of topics central to modern commutative algebra:

- the homological conjectures;
- problems in positive and mixed characteristic;
- tight closure and its interaction with birational geometry;
- integral dependence and blowup algebras;
- equisingularity theory;
- Hilbert functions and multiplicities;
- combinatorial commutative algebra;
- Gröbner bases and computational algebra.

Homological conjectures, positive and mixed characteristic, tight closure. Characteristic p methods, where p is a prime number, have been a powerful tool in both commutative algebra and algebraic geometry. Via reduction to characteristic p they also apply to the case of arbitrary characteristic. A culmination of this approach is the theory of tight closure conceived by Hochster and Huneke in the eighties, that led to powerful new results and sometimes strikingly simple proofs of known theorems. Very recently one of the main open problems in this area has been solved by Brenner and Monsky – the question of whether tight closure commutes with localization. Ein, Hara, Mustaţă, K. Smith, K. Watanabe and others have uncovered intriguing connections between tight closure and seemingly unrelated notions from birational geometry, such as rational singularities, multiplier ideals and arc spaces. These connections are being explored further, and have already led to deep results in both algebra and geometry.

The homological conjectures are a system of interrelated conjectures proposed by Hochster in the seventies that grew out of work of Serre, Peskine and Szpiro on intersection multiplicities. They have been solved for rings containing a field, but are open – and notoriously difficult – in the case of mixed characteristic. Fairly recently Heitmann was able to prove the direct summand conjecture in dimension three. His methods are currently being developed further by Lyubeznik, Roberts, Singh and others, who have uncovered deep properties of local cohomology modules with the aim of proving the conjectures in general.

Blowup algebras, integral dependence, equisingularity theory. There has been an abundance of new results about Rees algebras, or blowup algebras, and their structure over the past two decades. Rees algebras are the algebraic objects appearing in the process of resolving singularities, and they have been used in Kawasaki's celebrated proof of the existence of Macaulifications, a weak version of resolution

of singularities. Recent efforts, by Busé, Chardin, Cox, Jouanolou, Kustin, Polini, Simis, Ulrich, Vasconcelos and others, have focused on describing Rees algebras explicitly in terms of generators and relations, a problem known in the applied mathematics community as 'implicitation'.

Rees algebras are also the environment in which integral dependence of ideals and modules can be studied. Integral dependence is a notion of central importance in modern commutative algebra, and has been the subject of two recent monographs, one by Vasconcelos and one by Huneke and Swanson. The concept appears naturally in the study of multiplicities, Hilbert functions, cores and the aforementioned multiplier ideals. It also figures prominently in equisingularity theory, an area particularly well represented in Brazil.

Equisingularity theory strives to providing numerical criteria for when the members of a family of analytic sets are 'similar' to each other. Such criteria are provided by the equisingularity conditions of Whitney and Thom. Through work of Gaffney, Kleiman and Teissier these conditions are expressed in terms of integral dependence of Jacobian modules, which in turn can be translated into numerical conditions on multiplicities, at least in the case of isolated complete intersection singularities. In the light of recent progress in multiplicity theory one can hope that these results could be extended to more general types of singularities.

Combinatorial and computational commutative algebra, Gröbner bases, Hilbert functions. Since the seminal work of Hochster and Stanley in the seventies and eighties, combinatorial methods have played an important role in commutative algebra. The objects studied in this area include Stanley-Reisner rings of simplicial complexes, ideals of graphs, and toric varieties, which, being described by combinatorial data, are a useful testing group for statements about varieties in general. More recently, through work of Dickenstein, E. Miller and others, hypergeometric systems of linear partial differential equations have entered the realm of combinatorial commutative algebra as well.

A fundamental tool in computational algebra is Gröbner bases, a device that reduces arbitrary ideals to ideals generated by monomials, which are combinatorial objects and thereby more easily accessible computationally. Gröbner bases also play a role theoretically in the study of Hilbert functions, multiplicities and graded free resolutions. A recent breakthrough in this area has been the solution, by Eisenbud and Schreyer, of a longstanding conjecture relating the degree shifts in a free resolution to the multiplicity.

We end this Preface by expressing our gratitude to the contributors of this volume for their enthusiasm in the project. The anonymous referees, who worked very closely with us, also deserve special credit for all their time spent in reading and correcting the original manuscripts. We are aware of the many demands on our time that the academic profession requires from each of us! Finally, we wish to express our heartfelt gratitude to the following institutions for their generous financial support: CAPES, CNPq, Department of Energy, through its Office of Basic Energy Sciences, FACEPE (State Foundation of Pernambuco), National Science Foundation (NSF), through the grant NSF-OISE 0819049, Millenium Program, Third World Academy of Science (TWAS).

Alberto Corso and Claudia Polini
Lexington and Notre Dame · June 1, 2011

List of Mini Courses at PASI 2009

Elimination theory: Interactions between geometric modeling and commutative algebra
David Cox, Amherst College (USA)

Combinatorial commutative algebra
Jürgen Herzog, Universität Duisburg-Essen (Germany)

Tight closure theory and problems in positive characteristic
Craig Huneke, University of Kansas (USA)

Equisingularity theory
Steven Kleiman, Massachusetts Institute of Technology (USA)

Hilbert functions and Hilbert coefficients in local rings
Maria Evelina Rossi, Università di Genova (Italy)

Tropical algebra
Bernd Sturmfels, University of California at Berkeley (USA)

Rees algebras, integral closures and adjoint ideals
Bernd Ulrich, Purdue University (USA)

List of Talks at PASI 2009

Higher Fano manifolds
Carolina Araujo, IMPA (Brazil)

Reflexivity and rigidity for complexes
Luchezar Avramov, University of Nebraska (USA)

Implicit equations of multigraded hypersurfaces
Nicolás Botbol, Universidad de Buenos Aires (Argentina)

Normaliz: Algorithms for rational cones and affine monoids
Winfred Bruns, Universität Osnabrück (Germany)

Cremona geometry of plane curves
Ciro Ciliberto, Università di Roma (Italy)

Tropicalisation of rational varieties
Alicia Dickenstein, Universidad de Buenos Aires (Argentina)

Boji-Soederberg theory and the size of free resolutions
Daniel Erman, University of California at Berkeley (USA)

The Poincaré problem for subschemes invariant under Pfaff fields on projective spaces
Eduardo Esteves, IMPA (Brazil)

Danilov Gizatullin surfaces and G_a-actions
Hubert Flenner, Universität Bochum (Germany)

On certain maximal curves over finite fields
Arnaldo Garcia, IMPA (Brazil)

Gotzmann coefficients of Hilbert functions
Anthony Geramita, Queen's University (Canada)

On the hyperhomology of the small Gobelin for codimension 2
Xavier Gomez Mont, CIMAT (Mexico)

Cohen-Macaulayness versus vanishing of the first Hilbert coefficient of parameters: Towards a problem of Wolmer Vasconcelos
Shiro Goto, Meiji University (Japan)

Gonality of ACM curves in $\mathbb{P}^3$
Robin Hartshorne, University of California at Berkeley (USA)

A tight closure theory that commutes with localization in equal characteristic
Melvin Hochster, University of Michigan (USA)

A property of the ring of polynomials over a perfect field of characteristic $p > 0$
Gennady Lyubeznik, University of Minnesota (USA)

Blowup algebras and elimination theory
Claudia Polini, University of Notre Dame (USA)

Toric ideals of graphs and digraphs
Enrique Reyes, Instituto Politecnico Nacional/CINVESTAV (Mexico)

Oil fields and Hilbert schemes
Lorenzo Robbiano, Università di Genova (Italy)

Fontaine rings and local cohomology
Paul Roberts, University of Utah (USA)

Counting compatibly Frobenius split ideals
Karl Schwede, University of Michigan (USA)

Stanley decompositions of squarefree monomial ideals
YiHuang Shen, Purdue University (USA)

Local cohomology with determinantal support
Anurag Singh, University of Utah (USA)

Growth of Bass numbers
Janet Striuli, Fairfield University (USA)

Polynomial vector fields with algebraic trajectories
Israel Vainsencher, UFMG (Brazil)

On the ideal theory of graphs (fifteen years later)
Rafael H. Villarreal, Instituto Politecnico Nacional/CINVESTAV (Mexico)

The a-invariants of normal graded Gorenstein rings and varieties with even canonical class
Kei-ichi Watanabe, Nihon University (Japan)

PASI 2009 Photos

PHOTO 1. Mini course lecturers with Wolmer Vasconcelos

PHOTO 2. PASI 2009 organizers with Wolmer Vasconcelos

Photo 3. Group photo

This volume contains papers based on presentations given at the Pan-American Advanced Studies Institute (PASI) on commutative algebra and its connections to geometry, which was held August 3–14, 2009, at the Universidade Federal de Pernambuco in Olinda, Brazil.

The main goal of the program was to detail recent developments in commutative algebra and interactions with such areas as algebraic geometry, combinatorics and computer algebra. The articles in this volume concentrate on topics central to modern commutative algebra: the homological conjectures, problems in positive and mixed characteristic, tight closure and its interaction with birational geometry, integral dependence and blowup algebras, equisingularity theory, Hilbert functions and multiplicities, combinatorial commutative algebra, Gröbner bases and computational algebra.

ISBN 978-0-8218-4959-0

9 780821 849590

CONM/555