CONTEMPORARY MATHEMATICS

559

Tomography and Inverse Transport Theory

International Workshop on Mathematical Methods in Emerging Modalities of Medical Imaging October 25–30, 2009, Banff, Canada

International Workshop on Inverse Transport Theory and Tomography May 16–21, 2010, Banff, Canada

> Guillaume Bal David Finch Peter Kuchment John Schotland Plamen Stefanov Gunther Uhlmann Editors



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Dedicated to the memory of Professor Leon Ehrenpreis

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Preface

The volume contains research and review articles written by participants of two related international workshops "Mathematical Methods in Emerging Modalities of Medical Imaging" (October 25–30, 2009, see http://www.birs.ca/events/2009/5-day-workshops/09w5017) and "Inverse Transport Theory and Tomography" (May 16–21, 2010, see http://www.birs.ca/events/2010/5-day-workshops/10w5063) that took place at the Banff International Research Station. These workshops brought together 64 mathematicians, physicists, engineers, and medical researchers from 10 countries working at the cutting edge of medical, geophysics, and industrial imaging research and addressed the demanding mathematical problems arising in this fast developing area. Among the participants there were many leading experts, as well as graduate students, postdocs, and other junior researchers.

The research and survey articles, written for the volume, address cutting edge issues of the newly developing imaging modalities (photoacoustics, current impedance imaging, hybrid imaging techniques, elasticity imaging), as well as the recent progress in resolving outstanding problems of more traditional modalities, such as SPECT, ultrasound imaging, and inverse transport theory. Related topics of invisibility cloaking are also considered.

> The editors July 15, 2011

List of Workshop Participants

- M. Allmaras (Texas A&M University)
- G. Ambartsoumian (Univ. of Texas Arlington)
- S. Arridge (University College London)
- G. Bal (Columbia University)
- W. Bangerth (Texas A&M University)
- J. Boman (Stockholm University, Sweden)
- A. Bukhgeim (Wichita State University)
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- S. Carney (University of Illinois Urbana-Champaign)
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- N. Hoell (Columbia University)
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- Y. Hristova (IMA, Minneapolis)
- M. Hubenthal (University of Washington)
- D. Isaacson (Rensselaer Polytechnic Institute)
- A. Jollivet (Columbia University)
- A. Katsevich (University of Central Florida)
- A. Kim (University of California Merced)
- P. Kuchment (Texas A&M University)
- L. Kunyansky (University of Arizona, Tucson)
- Y. Kurylev (University College London, UK)
- I. Langmore (Columbia University)
- M. Lassas (University of Helsinki, Finland)
- A. Lawrence (University of California, San Diego)
- M. Lewis (University of Texas Southwestern Medical Center at Dallas)
- C. Li (Washington University)
- H. Liu (University of Washington)
- M. Machida (University of Pennsylvania)
- A. Malcolm (Massachusetts Institute of Technology)
- A. Manduca (Mayo Clinic)
- V. Markel (University of Pennsylvania)

- J. McLaughlin (Rensselaer Polytechnic Institute)
- S. Moskow (Drexel University)
- F. Monard (Columbia University)
- A. Nachman (University of Toronto)
- G. Nakamura (Hokkaido University, Japan)
- F. Natterer (University of Münster, Germany)
- L. V. Nguyen (University of Idaho)
- V. Palamodov (Tel Aviv University, Israel)
- S. Patch (University of Wisconsin, Milwaukee)
- E. T. Quinto (Tufts University)
- K. Ren (University of Texas at Austin)
- E. Ritman (Mayo Clinic College of Medicine)
- W. Rundell (Texas A&M University)
- M. Salo (University of Helsinki, Finland)
- O. Scherzer (University of Vienna, Austria)
- J. Schotland (University of Pennsylvania)
- P. Stefanov (Purdue University)
- A. Tamasan (University of Central Florida)
- T. Tarvainen (University of Kuopio, Finland)
- G. Uhlmann (University of Washington)
- L. V. Wang (Washington University, St Louis)
- Y. Xu (Ryerson University, Toronto, Canada)
- R. Zemp (University of Alberta, Canada)
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- T. Zhou (University of Washington)
- J.-R. Yoon (Clemson University)

This volume contains research and review articles written by participants of two related international workshops "Mathematical Methods in Emerging Modalities of Medical Imaging" (October 2009) and "Inverse Transport Theory and Tomography" (May 2010), which were held at the Banff International Research Station in Banff, Canada. These workshops brought together mathematicians, physicists, engineers, and medical researchers working at the cutting edge of medical imaging research and addressed the demanding mathematical problems arising in this area.

The articles, written by leading experts, address important analytic, numerical, and physical issues of the newly developing imaging modalities (e.g., photoacoustics, current impedance imaging, hybrid imaging techniques, elasticity imaging), as well as the recent progress in resolving outstanding problems of more traditional modalities, such as SPECT, ultrasound imaging, and inverse transport theory. Related topics of invisibility cloaking are also addressed.



