CONTEMPORARY MATHEMATICS

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Compact Moduli Spaces and Vector Bundles

Conference on Compact Moduli and Vector Bundles
October 21–24, 2010
University of Georgia
Athens, Georgia

Valery Alexeev
Angela Gibney
Elham Izadi
János Kollár
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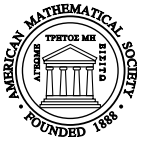

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Contents

Preface

On October 21-24, 2010, a conference on "Compact Moduli and Vector Bundles" was held at the University of Georgia in Athens, Georgia, USA. It was organized by Valery Alexeev, Angela Gibney, Elham Izadi, and David Swinarski, and supported by the University of Georgia and the National Science Foundation.

The main two topics of the conference were: compact moduli of varieties, or pairs of those, of arbitrary dimension, and conformal block bundles.

Among the geometrically meaningful compactifications of a moduli space of algebraic varieties, the one of Deligne-Mumford-Knudsen of pointed stable curves is perhaps the best known and studied. It found important applications in many fields, from number theory to theoretical physics.

In 1989 Kollár and Shepherd-Barron proposed a generalization of the notion of a stable curve to surfaces and higher-dimensional varieties, which was extended in 1996 by Alexeev to the case of pairs. Since then, many mathematicians, among them Alexeev, Hacking, Hassett, Keel, Kollár, Kovács, Pardini, Tevelev, van Opstall and Viehweg, worked on the details of the construction of the associated moduli space and analyzed specific examples.

Regarding the first topic, the aim of the conference was to review progress since the meeting on this subject in 2004 at the American Institute of Mathematics, Palo Alto. Although some of the talks were devoted to moduli spaces of stable pointed curves, the emphasis was on the higher-dimensional case.

The second major topic was vector bundles on compact moduli spaces, and in particular the conformal block bundles. These are certain vector bundles on $\overline{\mathrm{M}}_{g,n}$ which were defined by physicists in the 1970s. The input data for a conformal block bundle $\mathbb{V}_{\mathfrak{g}}(\ell; \lambda_1, \ldots, \lambda_n)$ are a semisimple Lie algebra $\mathfrak{g}$, an integer ℓ called the level or charge, and a collection of n irreducible representations λ_i of $\mathfrak{g}$. The rank of $\mathbb{V}_{\mathfrak{g}}(\ell; \lambda_1, \ldots, \lambda_n)$ is given by the Verlinde formula, which was proved mathematically in the 1990s by Beauville-Laszlo-Sorger, Faltings, Szenes, Labourie, and others. The work of Fakhruddin, appearing in this volume, takes this a step further by computing their Chern classes on $\overline{\mathrm{M}}_{g,n}$.

One may ask if analogous bundles can be defined on the moduli of higher-dimensional stable varieties and pairs. In a different direction, one can try to apply this new information to the study of the geometry of $\overline{\mathrm{M}}_{g,n}$. For example, it turns out that in genus 0 all conformal block bundles are globally generated. Thus, potentially, they may have applications to the F-conjecture of Fulton and Faber, which says that the cone of effective curves on $\overline{\mathrm{M}}_{0,n}$ is generated by closures of 1-dimensional strata.

The present volume aims to share the spirit and direction of the conference, rather than being a faithful record. Several of the speakers did not contribute, and

for those who did, the paper did not have to closely follow the talk. The editors were also happy to obtain contributions from several experts in the field who did not participate in the conference itself. As a consequence, we feel that the resulting volume gives a better idea of these rapidly changing fields.

Talks at the UGA conference

The conference had about a hundred participants, most of them graduate students and recent PhDs. There were twenty speakers. Below is the list of the talks that they delivered.

Thursday, October 21

- János Kollár (Princeton University), Moduli of varieties of general type: an overview.
- Sean Keel (University of Texas), Theta Functions for affine log K3 surfaces.
- Rita Pardini (Universita de Pisa), Abelian covers and complete moduli of varieties of general type.
- Radu Laza (SUNY Stonybrook), Moduli spaces birational to locally symmetric varieties of orthogonal or unitary type.

Friday, October 22

- James McKernan (MIT), Birational boundedness.
- Gavril Farkas (Humboldt University), Explicit parametrizations of moduli spaces of theta-characteristics.
- Ana-Maria Castravet (University of Arizona), Rigid curves on moduli spaces of stable rational curves.
- Jenia Tevelev (University of Massachusetts), Effective divisors on $\overline{M}_{0,n}$.
- David Hyeon (Postech), GIT and MMP for moduli of curves.

Saturday, October 23

- Shigeru Mukai (RIMS, Kyoto University), Moduli of Enriques surfaces of type E_7 and complete conics.
- Hélène Esnault (Essen University), Hodge theory and rational points.
- Yongnam Lee (Sogang University), Simply connected surfaces of general type with vanishing geometric genus in positive characteristic via deformation theory.
- Eduard Looijenga (Universiteit Utrecht), KZ systems and Hodge theory.
- Najmuddin Fakhruddin (Tata Institute, Mumbai), Chern classes of conformal blocks.
- Sándor Kovács (University of Washington), Vanishing theorems for log canonical pairs.

Sunday, October 24

- Nick Shepherd-Barron (Cambridge University), The local structure of the theta-null divisor on M_g.
- Jun Li (Stanford University), Hilbert-Mumford criterion for nodal curves.

- Paul Hacking (University of Massachusetts), Exceptional vector bundles associated to degenerations of surfaces.
- Barbara Fantechi (SISSA, Trieste), Compactifications via blowups: an overview.
- Brendan Hassett (Rice University), Lagrangian planes on holomorphic symplectic varieties.

Selected Titles in This Series

564 **Valery Alexeev, Angela Gibney, Elham Izadi, János Kollár, and Eduard Looijenga, Editors,** Compact Moduli Spaces and Vector Bundles, 2012

563 **Primitivo B. Acosta-Humánez, Federico Finkel, Niky Kamran, and Peter J. Olver, Editors,** Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics, 2012

562 **P. Ara, K. A. Brown, T. H. Lenagan, E. S. Letzter, J. T. Stafford, and J. J. Zhang, Editors,** New Trends in Noncommutative Algebra, 2012

561 **Óscar Blasco, José A. Bonet, José M. Calabuig, and David Jornet, Editors,** Topics in Complex Analysis and Operator Theory, 2012

560 **Weiping Li, Loretta Bartolini, Jesse Johnson, Feng Luo, Robert Myers, and J. Hyam Rubinstein, Editors,** Topology and Geometry in Dimension Three, 2011

559 **Guillaume Bal, David Finch, Peter Kuchment, John Schotland, Plamen Stefanov, Gunther Uhlmann, and Gunther Uhlmann, Editors,** Tomography and Inverse Transport Theory, 2011

558 **Martin Grohe and Johann A. Makowsky, Editors,** Model Theoretic Methods in Finite Combinatorics, 2011

557 **Jeffrey Adams, Bong Lian, and Siddhartha Sahi, Editors,** Representation Theory and Mathematical Physics, 2011

556 **Leonid Gurvits, Philippe Pébay, J. Maurice Rojas, and David Thompson, Editors,** Randomization, Relaxation, and Complexity in Polynomial Equation Solving, 2011

555 **Alberto Corso and Claudia Polini, Editors,** Commutative Algebra and Its Connections to Geometry, 2011

554 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex Analysis and Dynamical Systems IV, 2011

553 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex Analysis and Dynamical Systems IV, 2011

552 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the Quantum II, 2011

551 **Jesus Araujo-Gomez, Bertin Diarra, and Alain Escassut, Editors,** Advances in Non-Archimedean Analysis, 2011

550 **Y. Barkatou, S. Berhanu, A. Meziani, R. Meziani, and N. Mir, Editors,** Geometric Analysis of Several Complex Variables and Related Topics, 2011

549 **David Blázquez-Sanz, Juan J. Morales-Ruiz, and Jesús Rodríguez Lombardero, Editors,** Symmetries and Related Topics in Differential and Difference Equations, 2011

548 **Habib Ammari, Josselin Garnier, Hyeonbae Kang, and Knut Sølna, Editors,** Mathematical and Statistical Methods for Imaging, 2011

547 **Krzysztof Jarosz, Editor,** Function Spaces in Modern Analysis, 2011

546 **Alain Connes, Alexander Gorokhovsky, Matthias Lesch, Markus Pflaum, and Bahram Rangipour, Editors,** Noncommutative Geometry and Global Analysis, 2011

545 **Christian Houdré, Michel Ledoux, Emanuel Milman, and Mario Milman, Editors,** Concentration, Functional Inequalities and Isoperimetry, 2011

544 **Carina Boyallian, Esther Galina, and Linda Saal, Editors,** New Developments in Lie Theory and Its Applications, 2011

543 **Robert S. Doran, Paul J. Sally, Jr., and Loren Spice, Editors,** Harmonic Analysis on Reductive, p-adic Groups, 2011

542 **E. Loubeau and S. Montaldo, Editors,** Harmonic Maps and Differential Geometry, 2011

541 **Abhijit Champanerkar, Oliver Dasbach, Efstratia Kalfagianni, Ilya Kofman, Walter Neumann, and Neal Stoltzfus, Editors,** Interactions Between Hyperbolic Geometry, Quantum Topology and Number Theory, 2011

CONM 564

Compact Moduli Spaces and Vector Bundles • Alexeev et al., Editors

This book contains the proceedings of the conference on Compact Moduli and Vector Bundles, held from October 21–24, 2010, at the University of Georgia.

This book is a mix of survey papers and original research articles on two related subjects: Compact Moduli spaces of algebraic varieties, including of higher-dimensional stable varieties and pairs, and Vector Bundles on such compact moduli spaces, including the conformal block bundles. These bundles originated in the 1970s in physics; the celebrated Verlinde formula computes their ranks.

Among the surveys are those that examine compact moduli spaces of surfaces of general type and others that concern the GIT constructions of log canonical models of moduli of stable curves.

The original research articles include, among others, papers on a formula for the Chern classes of conformal classes of conformal block bundles on the moduli spaces of stable curves, on Looijenga's conjectures, on algebraic and tropical Brill–Noether theory, on Green's conjecture, on rigid curves on moduli of curves, and on Steiner surfaces.

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