

CONTEMPORARY MATHEMATICS

587

Diophantine Methods, Lattices, and Arithmetic Theory of Quadratic Forms

International Workshop
Banff International Research Station
November 13–18, 2011
Banff, Alberta, Canada

Wai Kiu Chan
Lenny Fukshansky
Rainer Schulze-Pillot
Jeffrey D. Vaaler
Editors



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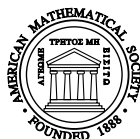
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American Mathematical Society
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Photograph of Boris Venkov courtesy of Martin Frick

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In Memoriam



Boris B. Venkov (1934 – 2011)

Boris Borisovich Venkov passed away on November 10, 2011, just 5 days before his 77th birthday. He was born in St. Petersburg (then Leningrad) in 1934. His father was the mathematician Boris Alekseevich Venkov, with whom he wrote two joint articles in 1962/1963. B. B. Venkov obtained his Ph.D. in 1952 under the direction of D. K. Faddeev, and since 1957 was a member of the Steklov Institute of Mathematics of the Academy of Sciences of the Soviet Union (later Russia) in St. Petersburg, where he became a Professor and Leading Research Fellow. After early work in the cohomology theory of finite groups, the study of lattices in Euclidean spaces became a central part of his work. One of his famous achievements in this subject was the introduction of spherical harmonic polynomials as a tool for the classification of even unimodular Euclidean lattices. During the past 30 years Venkov created a bridge between the theory of spherical designs and the theory of lattices in Euclidean spaces resulting in his very fruitful definition of strongly perfect lattices. In the years after 1990 he spent a great part of his mathematical life visiting various universities, mostly in France, Germany, Japan, and Switzerland, where he collaborated with several researchers. He was always full of original ideas and always eager to share and discuss them with others, thus inspiring many younger scientists.

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Preface

This volume of the AMS Contemporary Mathematics series contains the proceedings of the international workshop on Diophantine Methods, Lattices, and Arithmetic Theory of Quadratic Forms, held at the Banff International Research Station, Canada, November 13 - 18, 2011. The goal of this workshop was to bring together people working in the following areas:

1. Classical arithmetic and algebraic theory of quadratic forms and lattices.
2. Diophantine problems and the theory of height functions.
3. Extremal lattice theory and spherical designs.

In spite of the close connections between these areas, it is quite rare for mathematicians working in these subjects to meet altogether for a joint workshop.

The workshop was organized by W. K. Chan, L. Fukshansky, R. Schulze-Pillot, and J. Vaaler, who are also the editors of the current volume. There were 41 invited participants at the workshop, delivering a total of 6 plenary (hour long) and 16 invited (30 minute long) talks. The meeting was overshadowed by the sudden and unexpected death of Professor Boris B. Venkov in Aachen, Germany just days before the workshop. Venkov's important contributions to the theory of lattices and spherical designs played a central role at the conference. This volume is dedicated to the memory of Boris Venkov.

The current volume features 19 papers, 2 of which are surveys. In particular, the first article of this volume details Venkov's influential work on lattices and spherical designs. All the articles presented here have been rigorously refereed according to the high standards of publication required by the AMS Contemporary Mathematics series. The topics presented are well balanced to reflect the multiple themes discussed at the conference. We hope that this collection becomes a welcome addition to the existent literature on the subject.

We wish to thank the Banff International Research Station for the wonderful hospitality, as well as tremendous administrative, technical, and financial support in hosting this workshop. We would also like to thank the Number Theory Foundation, whose generous travel funding made it possible for junior participants to attend our workshop. Finally, we would like to thank the referees for their work in ensuring the high quality of this collection.

The editors
Wai Kiu Chan,
Lenny Fukshansky,
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October 2012

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This volume contains the proceedings of the International Workshop on Diophantine Methods, Lattices, and Arithmetic Theory of Quadratic Forms, held November 13–18, 2011, at the Banff International Research Station, Banff, Alberta, Canada.

The articles in this volume cover the arithmetic theory of quadratic forms and lattices, as well as the effective Diophantine analysis with height functions. Diophantine methods with the use of heights are usually based on geometry of numbers and ideas from lattice theory. The target of these methods often lies in the realm of quadratic forms theory. There are a variety of prominent research directions that lie at the intersection of these areas, a few of them presented in this volume:

- Representation problems for quadratic forms and lattices over global fields and rings, including counting representations of bounded height.
- Small zeros (with respect to height) of individual linear, quadratic, and cubic forms, originating in the work of Cassels and Siegel, and related Diophantine problems with the use of heights.
- Hermite's constant, geometry of numbers, explicit reduction theory of definite and indefinite quadratic forms, and various generalizations.
- Extremal lattice theory and spherical designs.

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