Hodge Theory,
Complex Geometry,
and Representation Theory

NSF-CBMS Regional Conference in Mathematics
Hodge Theory, Complex Geometry,
and Representation Theory
Texas Christian University, Fort Worth, Texas
June 18, 2012

Robert S. Doran
Greg Friedman
Scott Nollet
Editors
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This volume is dedicated to Phillip A. Griffiths.
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Preface

Professor Phillip Griffiths is a giant in complex geometry. He is best known for his use of transcendental methods in algebraic and differential geometry, and his many awards include the 1971 LeRoy P. Steele Prize for his fundamental work on period domains of abelian integrals and the 2008 Wolf Foundation Prize for his work on variations of Hodge structures. He has held various positions at Berkeley, Princeton, Harvard, and Duke and has been at the Institute for Advanced Study in Princeton since 1991, where he served as director from 1991 to 2003.

On June 18, 2012, Professor Griffiths was the plenary speaker at a Conference Board of the Mathematical Sciences (CBMS) conference hosted at Texas Christian University in Fort Worth, where he gave a series of 10 lectures on the topic “Hodge Theory, Complex Geometry, and Representation Theory”. In these lectures, Prof. Griffiths first explained now-classical work concerning how the structure of Shimura varieties as quotients of Mumford-Tate domains by arithmetic groups had been used to understand the relationship between Galois representations and automorphic forms; he then discussed how, due to a recent breakthrough of Carayol and a better understanding of cycle spaces, Penrose transforms, and Mumford-Tate domains, there is now the possibility of extending these results beyond the classical case. Despite the complexity of this topic, which occurs at the confluence of several major research areas in mathematics, the lectures were solidly grounded in key examples in order to make them accessible to a broad audience.

Prof. Griffiths’s conference lectures will be published as an independent book in the Conference Board of the Mathematical Sciences series of the American Mathematical Society. The papers contained in the present volume were contributed by other conference participants and include a balance between expository and research works, the topics all being heavily influenced by the work of Griffiths. The subjects of the expository papers include Noether-Lefschetz theory, algebraicity of Hodge loci, and the representation theory of $SL_2(\mathbb{R})$. The research articles concern the Hodge conjecture, Harish-Chandra modules, mirror symmetry, Hodge representations of $\mathbb{Q}$-algebraic groups, and the compactifications, distributions, and quotients of period domains.

The editors of this volume, who were also the conference organizers, wish to extend their deep gratitude to the National Science Foundation and the Conference Board of the Mathematical Sciences for making the conference possible via NSF grant DMS-1137952. We thank all of the participants who made for a lively week, and we especially thank those who contributed to this proceedings volume. We also acknowledge the hard work and help of the referees. Foremost, we thank Phillip Griffiths, whose work has served as an inspiration for all that can be found in these pages. Finally, we wish to thank Sergei Gelfand, Christine Thivierge, and the
dedicated staff of the American Mathematical Society for their efforts in publishing these proceedings.

Robert S. Doran
Greg B. Friedman
Scott R. Nollet

1The second-named editor was partially supported by a grant from the Simons Foundation (#209127 to Greg Friedman)

Not pictured: Jim Carlson, E. Javier Elizondo, Wushi Goldring, Loren Spice, Qiao Zhang
Conference Attendees

James Carlson  
University of Utah

Eduardo Cattani  
University of Massachusetts, Amherst

Xi Chen  
University of Alberta

Jeremy Daniel  
Université Paris Diderot

Bruce Doran  
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University of California, Los Angeles

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Tatsuki Hayama  
National Taiwan University

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Northeastern University

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Providence College

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CONICET, Argentina

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Washington University

Tyler Kelly  
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Amanda Knecht  
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Zhiyuan Li  
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University of Utah  

Sampei Usui  
Osaka University  

Jie Xia  
Columbia University  

Ke Xue  
University of Maryland, College Park  

Sen Yang  
Louisiana State University  

Yanhong Yang  
Columbia University  

Qiao Zhang  
Texas Christian University  

Zheng Zhang  
Stony Brook University  

Xiaolei Zhao  
University of Michigan  

Runpu Zong  
Princeton University
Conference Speakers

Phillip Griffiths
*Hodge Theory, Complex Geometry, and Representation Theory*

James Carlson
*Transcendence degree of the field of periods*

Eduardo Cattani
*Asymptotics of the Period Map*

Wushi Goldring
*Algebraicity of Automorphic Representations*

Mark Green
*Review of Real and Complex Compact and Semisimple Lie Groups and Finite Dimensional Representation Theory*

Aroldo Kaplan
*Topics from Griffiths’s Lecture 3*

Matt Kerr
*Representations of $SL_2$, parts I and II*

James Lewis
*Hodge Type Conjectures and the Bloch-Kato Theorem*

Gregory Pearlstein
*Boundary components of Mumford-Tate domains*

Colleen Robles
*Schubert integrals and invariant characteristic cohomology of the infinitesimal period relation*

Domingo Toledo
*Period Domains and Kähler Manifolds*

Sampei Usui
*Log Mixed Hodge Theory*
This volume contains the proceedings of an NSF/Conference Board of the Mathematical Sciences (CBMS) regional conference on Hodge theory, complex geometry, and representation theory, held on June 18, 2012, at the Texas Christian University in Fort Worth, TX. Phillip Griffiths, of the Institute for Advanced Study, gave 10 lectures describing now-classical work concerning how the structure of Shimura varieties as quotients of Mumford-Tate domains by arithmetic groups had been used to understand the relationship between Galois representations and automorphic forms. He then discussed recent breakthroughs of Carayol that provide the possibility of extending these results beyond the classical case. His lectures will appear as an independent volume in the CBMS series published by the AMS.

This volume, which is dedicated to Phillip Griffiths, contains carefully written expository and research articles. Expository papers include discussions of Noether-Lefschetz theory, algebraicity of Hodge loci, and the representation theory of $SL_2(\mathbb{R})$. Research articles concern the Hodge conjecture, Harish-Chandra modules, mirror symmetry, Hodge representations of $\mathbb{Q}$-algebraic groups, and compactifications, distributions, and quotients of period domains. It is expected that the book will be of interest primarily to research mathematicians, physicists, and upper-level graduate students.