Geometry and Topology of Submanifolds and Currents

2013 Midwest Geometry Conference (MGC XIX)
October 19, 2013
Oklahoma State University, Stillwater, OK

2012 Midwest Geometry Conference (MGC XVIII)
May 12–13, 2012
University of Oklahoma, Norman, OK

Weiping Li
Shihshu Walter Wei
Editors

American Mathematical Society
Geometry and Topology
of Submanifolds and Currents
Geometry and Topology of Submanifolds and Currents

2013 Midwest Geometry Conference (MGC XIX)
October 19, 2013
Oklahoma State University, Stillwater, OK

2012 Midwest Geometry Conference (MGC XVIII)
May 12–13, 2012
University of Oklahoma, Norman, OK

Weiping Li
Shihshu Walter Wei
Editors
Contents

Preface vii

2013 Midwest Geometry Conference (MGC XIX) Talks ix

Plateau Problems in Metric Spaces and Related Homology and Cohomology Theories
Robert M. Hardt 1

Relating Equivariant and Motivic Cohomology viaAnalytic Currents
Pedro F. dos Santos, Paulo Lima-Filho, and Robert M. Hardt 19

Braids and Symplectic Reidemeister Zeta Functions
Weiping Li 41

Systoles of Surfaces and 3-Manifolds
Lizhi Chen and Weiping Li 61

Ideal Theory and Classification on Isoparametric Hypersurfaces
Quo-Shin Chi 81

The Hartogs Triangle in Complex Analysis
Mei-Chi Shaw 105

Finite Volume Flows and Witten’s Deformation
Wenchuan Hu 117

On the Existence and Nonexistence of Stable Submanifolds and Currents in Positively Curved Manifolds and the Topology of Submanifolds in Euclidean Spaces
Ralph Howard and Shihshu Walter Wei 127

Remarks on Stable Minimal Hypersurfaces in Riemannian Manifolds and Generalized Bernstein Problems
Shihshu Walter Wei, Lina Wu, and Yongsheng Zhang 169
Preface

The selections in this volume are mainly from the 2013 Midwest Geometry Conference held at Oklahoma State University on October 19, 2013 and partly from the 2012 Midwest Geometry Conference held at The University of Oklahoma, May 12–13, 2012. The Midwest Geometry Conference was an annual event in the Midwest region from 1991 until 2007 and was revived in 2012 at The University of Oklahoma (cf. e.g. http://www.math.ou.edu/mgc20/history.php). The 2013 Midwest Geometry Conference focused on Plateau problems, equivariant motivic cohomology, ideal theory and classification of isoparametric hypersurfaces, stable submanifolds, calibrated geometry, p-harmonic geometry, and Dolbeault cohomology groups.

The first article in this volume gives a nice, brief introduction to Plateau problems in general and describes the recent result of Hardt with his collaborators T. De Pauw and W. Pfeffer. Hardt discusses the fundamental compactness and rectifiability and the applications to the Plateau problem and optimal transport paths. Dos Santos, Lima-Filho, and Hardt show possibly the best motivation for Voevodsky’s definition of motivic cohomology and apply it to singular and ordinary equivariant cohomology theories. Li shows that the Reidemeister number of a smooth map on the representation variety induced from the braid action provides a knot invariant of the corresponding braid, and the Reidemeister zeta function from the dynamic system point of view is a rational function for certain classes of braids.

Chen and Li present a survey on systolic inequalities and systolic freedom in two and three dimensions. They investigate the optimal systolic ratio and the realized metric for surfaces and discuss systolic inequalities and freedoms of homotopy, homology and stable and conformal systoles for 3-manifolds. Chi applies regular sequences, Cohen-Macaulayness, Serre’s criterion on reducedness, Serre’s criterion of primeness, Serre’s criterion of normality for homogeneous polynomials for his purpose of classifying isoparametric hypersurfaces of degree four, and reviews the classification theory with emphasis on the application of the ideal theory

Shaw gives an interesting review about $L^2$ theory and the regularity for $\overline{\partial}$ on the Hartogs triangle, and function theory for the related Hartogs triangle in the complex projective space $CP^2$. Hu provides an affirmative answer to a question raised by Harvey and Lawson on the direct connection between Witten’s deformation and finite volume flows.

It is one of the most fundamental and interesting problems in geometry to study the relationship between curvature and topology of Riemannian manifolds and Riemannian submanifolds. The famous Synge Theorem showed that a compact orientable even-dimensional Riemannian manifold with positive sectional curvature has no nonconstant stable closed geodesics; hence its fundamental group vanishes. Howard and Wei use an extrinsic average variational method in the calculus of
variations to discuss the nonexistence of stable $p$-currents on hypersurfaces in Euclidean spaces and submanifolds in space forms under certain curvature conditions. The nonexistence implies their $p$-th homology groups vanish and leads to classification theorems of stable rectifiable currents on the quaternionic projective spaces and the Cayle projective plane. It also leads to differentiable sphere theorems and classification theorems of homologically mass minimizing closed rectifiable currents in real projective spaces over $\mathbb{Z}_2$. They further prove the vanishing of $L^2$-harmonic 1-forms on noncompact stable minimal hypersurfaces, which leads to the vanishing of the first cohomology groups for the noncompact submanifolds.

Uniqueness in analysis, geometry and topology has been a very interesting, active and important area of research in mathematics. Wei, Wu, and Zhang extend some previous works on stable minimal hypersurfaces and generalized Bernstein type problems in noncompact Riemannian manifolds. They use a new approach by studying the interplay between the scalar curvature of the originally induced metric $g$ and the scalar curvature of a conformally changed metric with conformal function in $L^q(M, g)$ space where $q \in \mathbb{R}\backslash[0, 2)$.

We are most grateful for the financial support from the National Science Foundation. We thank all of our colleagues and students from the U.S. and abroad who participated in the conferences or contributed to this volume. We also acknowledge Oklahoma State University and The University of Oklahoma for assistance with the 2013 Midwest Geometry Conference. In particular, we wish to thank S. Downing and A. M. McFarlin at Oklahoma State University for their help at various stages of the conference, and Financial Coordinator Patricia E. Kimbrough at The University of Oklahoma for her professional service and help in financial management. A special thanks to Christine M. Thivierge, Associate Editor for Proceedings, the American Mathematical Society, for her superb organization and wonderful work in preparing this volume. Last but not least, we thank Mike Saitas, Production Editor, the American Mathematical Society, for making the present form and production of this volume possible.

Weiping Li
Shihshu Walter Wei
2013 Midwest Geometry Conference (MGC XIX) Talks

Robert M Hardt, Rice University, “Plateau Problems in Metric Spaces”

Yongsheng Zhang, MSRI, Berkeley and Northeast Normal University, China, “On Gluing Techniques in Calibrated Geometry”

Quo-Shin Chi, Washington University, “On the Classification of Isoparametric Hypersurfaces”

Shihshu Walter Wei, University of Oklahoma, “New Comparison Theorems, Conservation Laws, PDE, Cohomology and Sharp Estimates in p-Harmonic Geometry with Applications in Real and Complex World”

Gang Liu, University of California, Berkeley, “Hadamard Three Circle Theorem for Complete Kähler Manifolds with Applications”

Mei-Chi Shaw, University of Notre Dame, “Topology of Dolbeault Cohomology Groups”

Paulo Lima-Filho, Texas A&M University, “Real Analytic Cycles and Cycle Maps for the Motivic Cohomology of Real Varieties”
Selected Published Titles in This Series

646  Weiping Li and Shi Shu Walter Wei, Editors,  Geometry and Topology of Submanifolds and Currents, 2015

643  Tony Pantev, Carlos Simpson, Bertrand Toën, Michel Vaquié, and Gabriele Vezzosi, Editors,  Stacks and Categories in Geometry, Topology, and Algebra, 2015

642  Mustapha Lahyane and Edgar Martínez-Moro, Editors,  Algebra for Secure and Reliable Communication Modeling, 2015

641  Maria Basterra, Kristine Bauer, Kathryn Hess, and Brenda Johnson, Editors,  Women in Topology, 2015


639  C. S. Aravinda, William M. Goldman, Krishnendu Gongopadhyay, Alexander Lubotzky, Mahan Mj, and Anthony Weaver, Editors,  Geometry, Groups and Dynamics, 2015


637  Stéphane Ballet, Marc Perret, and Alexey Zaytsev, Editors,  Algorithmic Arithmetic, Geometry, and Coding Theory, 2015


635  Christopher W. Curtis, Anton Dzhamay, Willy A. Hereman, and Barbara Prinari, Editors,  Nonlinear Wave Equations, 2015

634  Steven Dougherty, Alberto Facchini, André Leroy, Edmund Puczyłowski, and Patrick Solé, Editors,  Noncommutative Rings and Their Applications, 2015


632  Gohar Kyureghyan, Gary L. Mullen, and Alexander Pott, Editors,  Topics in Finite Fields, 2015


630  Pierre Albin, Dmitry Jakobson, and Frédéric Rochon, Editors,  Geometric and Spectral Analysis, 2014


628  Anita T. Layton and Sarah D. Olson, Editors,  Biological Fluid Dynamics: Modeling, Computations, and Applications, 2014

627  Krishnaswami Alladi, Frank Garvan, and Ae Ja Yee, Editors,  Ramanujan 125, 2014


625  Alexander Barg and Oleg R. Musin, Editors,  Discrete Geometry and Algebraic Combinatorics, 2014


623  Pramod N. Achar, Dijana Jakelić, Kailash C. Misra, and Milen Yakimov, Editors,  Recent Advances in Representation Theory, Quantum Groups, Algebraic Geometry, and Related Topics, 2014

622  S. Ejaz Ahmed, Editor,  Perspectives on Big Data Analysis, 2014

For a complete list of titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/conmseries/.
The papers in this volume are mainly from the 2013 Midwest Geometry Conference, held October 19, 2013, at Oklahoma State University, Stillwater, OK, and partly from the 2012 Midwest Geometry Conference, held May 12–13, 2012, at the University of Oklahoma, Norman, OK.

The papers cover recent results on geometry and topology of submanifolds. On the topology side, topics include Plateau problems, Voevodsky’s motivic cohomology, Reidemeister zeta function and systolic inequality, and freedom in 2- and 3-dimensional manifolds. On the geometry side, the authors discuss classifying isoparametric hypersurfaces and review Hartogs triangle, finite volume flows, nonexistence of stable $p$-currents, and a generalized Bernstein type problem. The authors also show that the interaction between topology and geometry is a key to deeply understanding topological invariants and the geometric problems.