

# CONTEMPORARY MATHEMATICS

669

## Dynamics and Numbers

A Special Program: June 1–July 31, 2014

International Conference: July 21–25, 2014

Max-Planck Institute for Mathematics, Bonn, Germany

Sergiï Kolyada  
Martin Möller  
Pieter Moree  
Thomas Ward  
Editors



American Mathematical Society

# Dynamics and Numbers



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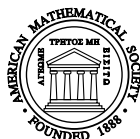
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## Preface

This volume contains the proceedings of the activity “Dynamics and Numbers” held at the Max-Planck Institute for Mathematics (MPIM) in Bonn, 1 June to 31 July 2014 and the conference on the same theme held in the final week of the activity. The activities brought together about a hundred researchers from 18 different countries and many of the long-term visitors at MPIM. Some of the papers in these proceedings reflect aspects of the collaborative work done during the activity. As with the proceedings of the activities on a similar theme in 2004 and 2009, which appeared as volumes 385 and 532 in the Contemporary Mathematics series, the talks presented at MPIM and the papers here reflect the vitality and diversity of research in dynamical systems, and the extent of the ongoing interaction between number theory and dynamical systems.





The topics of the activity covered dynamical systems and number theory in the broadest sense, instances of the interplay between them, and some applications to natural sciences. The main subjects included asymptotic geometric analysis and topological transformation groups; arithmetic dynamics; polynomials and pointwise ergodic theorems; actions of Polish groups; low-dimensional dynamics: graph theory, rotation theory, complex and real dynamics; interval exchange transformations and translation flows; billiards; symbolic dynamics; multifractal analysis and Diophantine approximation; Perron-Frobenius matrices, pseudo-Anosov maps, right-angled Artin groups and outer space; dynamics and moduli spaces; invariant measures and Littlewood's conjecture; Heisenberg odometers; dynamics and quasicrystals; dynamical systems of non-algebraic origins; flows on manifolds; translation surfaces and Abelian differentials, representations of integers; statistical properties of dynamical systems; topological orbit equivalence; and the theory of entropy and chaos.

The editors wish to record their thanks to the staff at the Max-Planck Institute, and to the many researchers who took part in the activity, for all their efforts in making this such a productive and enjoyable event.

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This volume contains a collection of survey and research articles from the special program and international conference on Dynamics and Numbers held at the Max-Planck Institute for Mathematics in Bonn, Germany in 2014.

The papers reflect the great diversity and depth of the interaction between number theory and dynamical systems and geometry in particular. Topics covered in this volume include symbolic dynamics, Bratelli diagrams, geometry of laminations, entropy, Nielsen theory, recurrence, topology of the moduli space of interval maps, and specification properties.

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