

CONTEMPORARY MATHEMATICS

673

Recent Developments in Representation Theory

Maurice Auslander Distinguished Lectures
and International Conference

May 1–6, 2014

Woods Hole Oceanographic Institute,
Woods Hole, MA

Alex Martsinkovsky

Gordana Todorov

Kiyoshi Igusa

Editors



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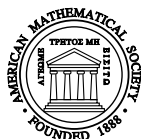
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Preface

This volume continues the newly established tradition of publishing expository lectures from the Maurice Auslander Distinguished Lectures and International Conference. These events took place May 1-6, 2014 at the Woods Hole Oceanographic Institute in Falmouth, MA.

The support from the National Science Foundation (Grant DMS-1162304) allowed us to invite 59 participants from Austria, Canada, Czech Republic, Germany, India, Mexico, Norway, Russia, Spain, Turkey, the UK, and the US. Twenty five of them were either graduate students or postdocs.

The talks in the conference spread over a wide area of research. The same applies to the selected expository lectures in this volume. Vladimir Bavula presents a survey of old and new results on left orders in left Artinian rings. Ellen Kirkman gives an introduction to invariant theory of Artin-Schelter regular algebras. Claus-Michael Ringel describes the latest results relating combinatorics and representation theory of hereditary artin algebras. Evgeny Smirnov's survey touches upon the classical topic of Schubert calculus on Grassmannian and flag varieties and relates it to the combinatorics of Gelfand-Zetlin polytopes. Luise Unger introduces the reader to the combinatorics of tilting modules.

The list of the topics above, as spread out as it is, does not fully represent the scientific breadth of the conference, even in the expository aspect of it. Because of the time constraints, some of the presented expository lectures did not transition into a written format. The editors hope very much that at least some of them will eventually make it into print.

The support for the Conference from the NSF and the multi-year continued support for the Maurice Auslander Distinguished Lectures from Bernice Auslander create a unique symbiotic relationship between the two events. The expository component of the Conference has been attracting a large number of graduate students and young researchers. The editors of this volume hope that its publication will benefit many more mathematicians around the world.

The Editors

SELECTED PUBLISHED TITLES IN THIS SERIES

- 673 **Alex Martsinkovsky, Gordana Todorov, and Kiyoshi Igusa, Editors**, Recent Developments in Representation Theory, 2016
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This volume contains selected expository lectures delivered at the Maurice Auslander Distinguished Lectures and International Conference, held May 1–6, 2014, at the Woods Hole Oceanographic Institute, Woods Hole, MA.

Several significant developments of the last decade in representation theory of finite-dimensional algebras are related to combinatorics. Three of the five lectures in this volume deal, respectively, with the Catalan combinatorics, the combinatorics of Gelfand-Zetlin polytopes, and the combinatorics of tilting modules. The remaining papers present history and recent advances in the study of left orders in left Artinian rings and a survey on invariant theory of Artin-Schelter regular algebras.

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