CONTEMPORARY MATHEMATICS

684

Categorification in Geometry, Topology, and Physics

Anna Beliakova
Aaron D. Lauda
Editors

American Mathematical Society

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10 9 8 7 6 5 4 3 2 1 22 21 20 19 18 17

Dedicated to Christian Blanchet
on the occasion of his sixtieth birthday

Contents

Preface

The emergent mathematical philosophy of categorification is reshaping our view of modern mathematics by uncovering a hidden layer of structure in mathematics, revealing richer and more robust structures capable of describing more complex phenomena. This philosophy has led to a number of shocking new results and has ushered in a new area of interaction between algebra, geometry, and theoretical physics. Categorification is a powerful tool for relating various branches of mathematics and exploiting the commonalities between fields. It provides a language emphasizing essential features and allowing precise relationships between vastly different fields.

Categorification exemplifies the duality between pure mathematics and theoretical physics, with ideas from one area inspiring innovation in the other. Key concepts are exchanged between fields and transported to other areas using the powerful descriptive ability enabled by categorification. Indeed, the term "categorification" originated in work of Crane and Frenkel who were studying techniques for extending 3-dimensional topological quantum field theories to 4-dimensions. This area has rapidly developed over the last twenty years far beyond these original considerations, blossoming into an exciting and dynamic area of modern mathematics, with deep relations to geometric representation theory, low-dimensional topology, algebraic geometry, and mathematical physics.

Classical structures in representation theory, such as simple Lie algebras and their representations, allow deformations into quantum groups and modules over them, as discovered by Drinfeld and Jimbo. In turn, quantum groups lead to a representation-theoretical interpretation of the Jones, Kauffman, HOMFLY-PT and other link polynomials. These theories are closely connected to physics via the Chern-Simons path integral, as shown by Witten in his seminal paper on the Jones polynomial. More recently, it became clear that quantum deformation was only a prelude into deeper structural beauty and meaning. In the work of Khovanov, Ozsvath, Szabo, Rasmussen, Rozansky, and many others, *quantum link invariants* were interpreted as Euler characteristics of various bigraded and multi-graded link homology theories. These theories carry four-dimensional information, providing invariants of link cobordisms. The resulting link homology theories appear to be a nexus between some of the most sophisticated directions in modern mathematics and theoretical physics.

The discovery of link homology gave rise to new advances in representation theory, as quantum groups and related algebras were categorified. The resulting higher representation theory created a new bridge between representation theory and the link homology theories, which despite the increasing number of publications in this area is still far from being fully explored.

This volume was inspired by the conference Categorification in Algebra, Geometry, and Physics (a conference in honor of the 60th Birthday of Christian Blanchet), which took place at IESC, Menasina, Cargese, France, from May 4th to May 8th, 2015. This conference made clear that categorification is a rapidly emerging area of intense study. It also elucidated the need for a reference for newcomers to the field to learn the types of tools used in categorification, the problems where these tools have been successfully applied, and the future directions in which the field is moving. Our aim with this volume is to address this need. To this end, we have solicited articles from experts in categorification from around the world who were invited to share their unique perspective.

This volume focuses on the role categorification plays in geometry, topology, and physics. These articles illustrate many important trends for the field including geometric representation theory, homotopical methods in link homology, interactions between higher representation theory and gauge theory, and DAHA approaches to link homology.

The organizers wish to thank the John Templeton Foundation and the University of Zurich for their generous support in making this conference possible.

Anna Beliakova
Aaron Lauda

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The companion volume (Contemporary Mathematics, Volume 683) is devoted to categorification and higher representation theory.

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