CONTEMPORARY MATHEMATICS

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New Directions in Homotopy Theory

Second Mid-Atlantic Topology Conference
March 12–13, 2016
Johns Hopkins University, Baltimore, Maryland

Nitya Kitchloo
Mona Merling
Jack Morava
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Preface

Introduction

This volume has grown out of the Second Mid-Atlantic Topology Conference, which was held in the Spring of 2016 in Baltimore[1]. The conference was motivated by the organizers' sense that algebraic topology, geometry, and category theory are in the midst of a generational shift of language, techniques, and paradigms, rooted in new thinking about higher homotopy-theoretic structures which profoundly alters our understanding of the nature of geometry and its applications across mathematics, reaching from data analysis to physics.

These fields are in ferment, with a forefront defined by a younger generation who bring to bear new perspectives from algebra on classical questions of pure and applied mathematics. The conference attempted to sample some of these developments, and to showcase some of the work of the research cohort with applications in fields such as

• arithmetic geometry (new aproaches to motives, e.g. via $\mathbb{A}^1$-homotopy theory; new techiques in K-theory via Hochschild and cyclic homology; homotopy-theoretic methods in automorphic forms, with applications to classical homotopy theory),

• geometric quantum field theory, e.g. through various cobordism hypotheses (via chiral homology and such, not necessarily E_∞ ring spectra), and

• category theory, e.g. through Voevodsky's univalent foundations project and related work on homotopy type theory by researchers drawn from algebraic topology, category theory, logic, and computer science (with more distantly related applications to topological data analysis).

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Emmy Noether shifted the focus of algebraic topology from numerical invariants of space (such as Betti numbers) to the algebraic objects (e.g. modules of geometric cycles) underlying them. More recently, interest has broadened from topological (homeomorphism) invariants to homotopy theory in general, with an attendant expansion of potential applications. This has extended even further, to issues of logic and foundations, so that homotopy types, rather than sets, are becoming central, fundamental objects of mathematical discourse.

[1]Financial support was provided by the Johns Hopkins University Department of Mathematics and the NSF, via grant DMS 1619569. The conference web page is at `http://www.math.jhu.edu/~eriehl/matc2016/` with a list of talk abstracts at

`http://www.math.jhu.edu/~eriehl/matc2016/abstracts.pdf`

As Yuri Manin puts it:

> I am pretty strongly convinced that there is an ongoing reversal in the collective consciousness of mathematicians: the homotopical picture of the world becomes the basic intuition, and if you want to get a discrete set, then you pass to the set of connected components of a space defined only up to homotopy. ... Cantor's problems of the infinite recede to the background: from the very start, our images are so infinite that if you want to make something finite out of them, you must divide them by another infinity. — Yuri Manin, "We do not choose mathematics as our profession. It chooses us: Interview with Yuri Manin" by Mikhail Gelfand

The papers in this volume are all concerned with homotopy theory in this global sense. Tabuada, for example, surveys recent developments in algebraic geometers' theories of motives — for example, generalizations to categories of (not necessarily commutative) differential graded algebras. Heller and Ormsby, working with more classical motives defined over the field of real numbers, show that their associated spectra of complex points behave well, when regarded as genuinely $\mathbb{Z}/2$-equivariant spectra, while Kass and Wickelgren show that related questions about spectra associated to arithmetic objects (i.e. defined over number fields) conceal further mysteries.

Delicate questions of equivariant homotopy theory lurk behind another subset of papers, centered around more general categorical questions. Lind and Malkiewich on one hand, and Ponto and Shulman on another, bring categories of diagrams (or, parametrized families) of spectra, and their invariants, under firm control. Szymik extends (and clarifies) the notion of categorical center to the homotopy-theoretic contexts, while Zakharevich exhibits a closed structure on the multicategory of Waldhausen categories. Such work takes us "behind the homotopy groups", enabling us to work more directly with the geometric questions that engaged us in the first place.

A final circle of papers is concerned with hard computational questions about just how well constructions from algebraic geometry (moduli spaces of elliptic curves or formal groups) model phenomena in chromatic homotopy theory. McTague shows that the spectrum of topological modular forms is not easily represented geometrically in terms of string bordism, and Peterson pins down some of the subtleties in the theory of orientations for higher analogs of MString. Kitchloo, Lorman, and Wilson, working at the oddly difficult prime two, pioneer the study of higher $\mathbb{Z}/2$-equivariant chromatic theory.

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Nitya Kitchloo
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New Directions in Homotopy Theory • Merling et al., Editors

This volume contains the proceedings of the Second Mid-Atlantic Topology Conference, held from March 12–13, 2016, at Johns Hopkins University in Baltimore, Maryland.

The focus of the conference, and subsequent papers, was on applications of innovative methods from homotopy theory in category theory, algebraic geometry, and related areas, emphasizing the work of younger researchers in these fields.

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