Representation Theory and Harmonic Analysis on Symmetric Spaces

AMS Special Session in Honor of Gestur Ólafsson’s 65th Birthday
Harmonic Analysis
January 4, 2017
Atlanta, Georgia

Jens Gerlach Christensen
Susanna Dann
Matthew Dawson
Editors
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JOSEPH A. WOLF
Preface

Dating back to the original work of Fourier on the solution to the heat equation using trigonometric series and the subsequent work of 19th- and early 20-century mathematicians to put it on a rigorous foundation, harmonic analysis has been an active area of research whose fruitful connections to other branches of mathematics have steadily grown stronger and more numerous over time. This volume, for instance, features articles in the areas of integral geometry, complex analysis, operator algebras, Lie algebras, special functions, and differential operators. The breadth of contributions showcases the diversity of current research in harmonic analysis and shows that it continues to be a vibrant and fruitful field of inquiry.

The connection between representation theory and harmonic analysis on symmetric spaces is, in particular, emphasized by many of the articles in this volume. This relationship has a long history, including many classical and famous results by Gelfand, Helgason, Harish-Chandra and others which have allowed, for instance, the development of a very precise description of the decomposition of the unitary, quasiregular representation associated to a Riemannian symmetric space $G/K$ as a direct integral of so-called $K$-spherical representations of $G$ with respect to a Plancherel measure given in terms of the beautiful c-function of Harish-Chandra. Several decades after these fundamental results, the role played by representation theory in analysis on symmetric spaces continues to be successfully exploited, as exhibited by many of the articles we have collected here.

This volume grew out of an AMS Special Session at the Joint Mathematics Meeting in Atlanta, Georgia in January, 2017. This session was held in honor of Gestur Ólafsson on his 65th birthday, and celebrated his many and wide-ranging contributions to the area of harmonic analysis. The session drew experts from around the globe, and several have also contributed to this volume. Gestur Ólafsson has made important contributions to harmonic analysis on symmetric spaces, unitary representations of semisimple Lie groups, reflection positivity in quantum field theory (including its connections with analysis on symmetric spaces), and wavelet theory, to name a few areas.

As students of Gestur, we are immensely grateful for the guidance and leadership he has shown us over the years. We have always found him to be exceptionally generous with his time and ideas, both as a mentor and as a collaborator. With this volume we wish to say both “thank you” and “congratulations” to our mentor.

We wish to thank the participants of the special session for making it such a success, as well as the contributors to this volume, which we hope will serve as a motivation and point of entry for young researchers, as well as provide valuable information for experts in this field. Thank you also to the many referees, without whose valued opinions this would not have been possible. Last but not least, we
thank the American Mathematical Society for letting us organize the special session, as well as Chris Thivierge for her help during the preparation of this volume.
List of Speakers

AMS Special Session on Harmonic Analysis
(In Honor of Gestur Olafsson’s 65th Birthday) I & II
Joint Mathematics Meetings
Atlanta, GA
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Pierre Clare (Dartmouth College): Geometric pictures of intertwining operators.
Matthew Dawson (CIMAT, Mexico): Principal series representations of direct-limit groups.
Fulton B Gonzalez (Tufts University): Surjectivity of Mean Value Operators on Noncompact Symmetric Spaces.
Benjamin Harris (Bard College at Simon’s Rock): Vogan-Zuckerman Characters and Semisimple Coadjoint Orbits.
Markus Hunziker (Baylor University): The Tricomi equation and complementary series representations of SL(2, \(\mathbb{R}\)).
Toshiyuki Kobayashi (The University of Tokyo): Conformally Covariant Symmetry Breaking Operators on Differential Forms and Some Applications.
Sergii Myroshnychenko (Kent State University): On polytopes with congruent projections or sections.
Angela Pasquale (Université de Lorraine): Radial parts of differential operators and a one-parameter family of hypergeometric functions of type BC.
Raul Quiroga-Barranco (CIMAT, Mexico): Multiplicity-free restrictions of holomorphic discrete series and Toeplitz operators.
Boris Rubin (Louisiana State University): New Inversion Formulas for the Horospherical Transform.
Siddhartha Sahi (Rutgers University): Macdonald hypergeometric functions.
Henrik Schlichtkrull (University of Copenhagen, Denmark): Real spherical spaces and their classification.
Robert J. Stanton (Ohio State University): Duality of geometries on flag manifolds associated to maximal parabolic subgroups in split G2.
Bent Ørsted (Aarhus University, Denmark): Homomorphisms between Verma modules and small representations.
Participants in the special session

From left to right: Raúl Quiroga-Barranco, Pierre Clare, Boris Rubin, Mark Sepanski, Markus Hunziker, Angela Pasquale, Susanna Dann, Toshiyuki Kobayashi, Matthew Dawson, Gestur Ólafsson, Sergii Myroshnychenko, Siddharta Sahi, Henrik Schlichtkrull, Ben Harris, Bent Ørsted, Robert Donley, Robert Stanton. Jens Christensen, Fulton Gonzalez

Photo courtesy of Gestur Ólafsson
Gestur Ólafsson’s contributions to Mathematics

Books


Refereed articles and refereed chapters in books and special volumes


(49) Continuous action of Lie groups on $\mathbb{R}^n$ and frames. *International Journal of Wavelets, Multiresolution and Information Processing* 3 No. 2 (2005), 211–235.


(74) (with A. Pasquale) The Cos\(^\lambda\) and Sin\(^\lambda\) transforms as intertwining operators between generalized principal series representations of SL\((n + 1, \mathbb{K})\). *Advances in Mathematics* **229** (2012), 267–293.


Book reviews


Non refereed chapters in books, conference proceedings and other publications

(1) Several publications in *Mathematica Gottingensis*, publication of the Sonderforschungsbereich Geometry and Analysis at the University of Göttingen and material posted on my webpage.


(10) Fourier-greining á víxhnum grúpum. Verpíll 2009, 34–36. (Verpíll is a journal for students in mathematics and physics at the University of Iceland. The English translation of the title is: Fourier analysis on Abelian groups.)


Graduate Students


(6) M. Dobrescu (Louisiana State University, 2005): Wavelet Sets with and without Groups and Multiresolution Analysis.

(7) M. Aristidou (Louisiana State University, 2005): Laguerre Functions Associated to Euclidean Jordan Algebras.

(8) K. Wiboonton (Louisiana State University, 2009): The Segal-Bargmann Transform on Inductive Limits of Compact Symmetric Spaces.

(9) J. Christensen (Louisiana State University, 2009): Function Spaces, Wavelets and Representation Theory.

(10) S. Dann (Louisiana State University, 2011): Paley-Wiener Theorems with Respect to the Spectral Parameter.


(13) C. A. Cross (Louisiana State University, 2015): Partial Cosine-Funk Transforms at Poles of the Cosine-$\lambda$ Transform on Grassmann Manifolds.

(15) M. Hayajneh (Louisiana State University, 2016): *Twisted Reflection Positivity*.

**Postdoctoral Scholars**

(1) Andreas Neumann (Louisiana State University, 1998–2000)
(2) Shijun Zheng (Louisiana State University, 2003–2005)
(3) Benjamin Harris (Louisiana State University, 2011–2014)
(4) Eli Vanney Roblero-Mendez (Louisiana State University, 2014)
(5) Stéphane Merigon (Louisiana State University, 2014)
Selected Published Titles in This Series

714 Jens Gerlach Christensen, Susanna Dann, and Matthew Dawson, Editors, Representation Theory and Harmonic Analysis on Symmetric Spaces, 2018

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This volume contains the proceedings of the AMS Special Session on Harmonic Analysis, in honor of Gestur Ólafsson’s 65th birthday, held on January 4, 2017, in Atlanta, Georgia.

The articles in this volume provide fresh perspectives on many different directions within harmonic analysis, highlighting the connections between harmonic analysis and the areas of integral geometry, complex analysis, operator algebras, Lie algebras, special functions, and differential operators. The breadth of contributions highlights the diversity of current research in harmonic analysis and shows that it continues to be a vibrant and fruitful field of inquiry.