

# CONTEMPORARY MATHEMATICS

723

## New Developments in the Analysis of Nonlocal Operators

AMS Special Session  
New Developments in the  
Analysis of Nonlocal Operators  
October 28–30, 2016  
University of St. Thomas, Minneapolis, Minnesota

Donatella Danielli  
Arshak Petrosyan  
Camelia A. Pop  
Editors

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2010 *Mathematics Subject Classification*. Primary 35R09, 35R11, 35R35;  
Secondary 11M41, 26A33, 60G51, 91G80.

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### Library of Congress Cataloging-in-Publication Data

Names: AMS Special Session on New Developments in the Analysis of Nonlocal Operators (2016: Minneapolis, Minn.) | Danielli, Donatella, 1966– editor. | Petrosyan, Arshak, 1975– editor. | Pop, Camelia A., 1983– editor.

Title: New developments in the analysis of nonlocal operators: AMS Special Session on New Developments in the Analysis of Nonlocal Operators, October 28–30, 2016, University of St. Thomas, Minneapolis, Minnesota / Donatella Danielli, Arshak Petrosyan, Camelia A. Pop, editors.

Description: Providence, Rhode Island: American Mathematical Society, [2019] | Series: Contemporary mathematics; volume 723 | Includes bibliographical references.

Identifiers: LCCN 2018039664 | ISBN 9781470441104 (alk. paper)

Subjects: LCSH: Operator theory–Congresses. | Functional analysis–Congresses. | Differential equations–Congresses.

Classification: LCC QA329 .A49 2019 | DDC 515/.724–dc23

LC record available at <https://lcn.loc.gov/2018039664>

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: <https://doi.org/10.1090/conm/723>

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10 9 8 7 6 5 4 3 2 1      24 23 22 21 20 19

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## Preface

Over the last decade there has been a resurgence of interest in nonlocal operators. The distinctive feature of such operators, and of the associated equations, is that (unlike in the case of classical partial differential equations) the behavior of the solution at a point depends not only on the behavior of the function nearby but also on the values of the function far away. To a great extent, the study of nonlocal equations is motivated by applications. For instance, fully nonlinear integro-differential equations naturally arise in the study of certain problems in stochastic control. Another prime example of a nonlocal operator of elliptic type is the fractional Laplacian. In turn, the study of nonlocal operators has led to the development of a wide range of new mathematical tools and methods, and much progress has been made by researchers working in different areas.

This volume contains papers contributed by speakers and participants of the Special Session on New Developments in the Analysis of Nonlocal Operators, AMS Sectional Meeting at the University of St. Thomas, Minneapolis, Minnesota, October 28–30, 2016. The aim of the special session was to stimulate interaction on the latest developments of analytic, geometric, and probabilistic methods for problems involving nonlocal operators. The volume starts with the paper “Fractional thoughts” by N. Garofalo, which provides a comprehensive introduction to various aspects of the fractional Laplacian (also spelled Laplacean), with many historical remarks and an extensive and up-to-date list of references, suitable for the beginners and more seasoned researchers alike. M. Allen’s paper proves uniqueness for weak solutions to abstract parabolic equations with the fractional Marchaud or Caputo time derivatives. H. Chang-Lara and O. Savin study the behavior of the one-phase Bernoulli free boundary problem near a fixed boundary by relating to a Signorini-type obstacle problem, which in turn is related to an obstacle problem for a half Laplacian. P. L. De Nápoli and P. R. Stinga reveal connections between fractional powers of the spherical Laplacian and functions from the analytic number theory and differential geometry such as Hurwitz zeta function and the Minakshisundaram zeta function. The volume concludes with a paper by the three of us on obstacle problems for a class of not stable-like nonlocal operators that include non-Gaussian asset price models widely used in mathematical finance, such as Variance Gamma Processes and Regular Lévy Processes of Exponential type.

We hope you will enjoy this volume!

Donatella Danielli  
Arshak Petrosyan  
Camelia A. Pop





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This volume contains the proceedings of the AMS Special Session on New Developments in the Analysis of Nonlocal Operators, held from October 28–30, 2016, at the University of St. Thomas, Minneapolis, Minnesota.

Over the last decade there has been a resurgence of interest in problems involving nonlocal operators, motivated by applications in many areas such as analysis, geometry, and stochastic processes.

Problems represented in this volume include uniqueness for weak solutions to abstract parabolic equations with fractional time derivatives, the behavior of the one-phase Bernoulli-type free boundary near a fixed boundary and its relation to a Signorini-type problem, connections between fractional powers of the spherical Laplacian and zeta functions from the analytic number theory and differential geometry, and obstacle problems for a class of not stable-like nonlocal operators for asset price models widely used in mathematical finance.

The volume also features a comprehensive introduction to various aspects of the fractional Laplacian, with many historical remarks and an extensive list of references, suitable for beginners and more seasoned researchers alike.



ISBN 978-1-4704-4110-4



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CONM/723