CONTEMPORARY MATHEMATICS

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Homotopy Theory: Tools and Applications

A Conference in Honor of Paul Goerss's 60th Birthday
July 17–21, 2017
University of Illinois at Urbana-Champaign
Urbana, Illinois

Daniel G. Davis
Hans-Werner Henn
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The editors dedicate this volume to Paul Goerss
on the occasion of his 60th birthday.

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Preface

This volume of articles is the proceedings for the conference *Homotopy Theory: Tools and Applications* hosted July 17-21, 2017 by the Department of Mathematics of the University of Illinois at Urbana-Champaign, and organized by Daniel Davis, Mark W. Johnson, Charles Rezk, and Vesna Stojanoska. This conference included over 150 participants representing institutions from a dozen countries, and consisted of 20 one-hour plenary lectures along with 24 half-hour contributed talks organized into four parallel sessions. Lists of these talks appear below. Conference participants were invited to submit original research articles, which all went through a standard refereeing process.

The original impetus for organizing the conference was the 60th birthday of Paul Goerss, and the breadth of the articles included here is a reflection of his broad interests within homotopy theory. Paul began his career working with Brown-Gitler spectra. His early Astérisque volume on Andre-Quillen cohomology set the stage for his later important work with Mike Hopkins on moduli problems for structured ring spectra, culminating in the Hopkins-Miller Theorem.[1] His work on homotopy fixed points foreshadowed his more recent important work with resolutions in chromatic homotopy theory and the chromatic splitting conjecture. In addition, his influential Bourbaki survey on topological modular forms is a fine example of his clear expository style.

Paul has also had an impact on how homotopy theory is passed on to the succeeding generation of researchers. In addition to his broad influence through the now standard reference *Simplicial Homotopy Theory*, he has advised 20 Ph.D. students so far, and he has organized a variety of major events. Among these are 5 Oberwolfach meetings (including a spring 2019 Arbeitsgemeinschaft), 3 Fields Institute meetings, and 2 events at the Newton Institute (including a program throughout the second half of 2018).

Now we share a morsel concerning each of the articles included here:

- *Beaudry:* locates the α-family in $\pi_*(L_{K(2)}S^0)$, for the difficult $p = 2$ case, thereby aiding the effort to understand $\pi_*(L_{K(2)}S^0)$ through duality spectral sequences.
- *Blanc, Johnson, and Turner:* provides a very general definition of a higher homotopy operation as an obstruction to rigidifying a homotopy commutative diagram, valid even in unpointed situations.
- *Blumberg and Hill:* details illustrative examples of computations as well as providing an overview of Tambara functors, or the natural structures on the zeroth homotopy groups of algebras over N_∞ operads, the truly equivariant versions of E_∞ operads.

[1] Some authors refer to this as the Goerss-Hopkins-Miller Theorem, but Paul does not.

- *Henn:* constructs algebraic centralizer resolutions for the Morava stabilizer group $\mathbb{G}_2$ and for the related subgroups $\mathbb{S}_2^1$ and $\mathbb{G}_2^1$, as well as topological realizations of them, thus providing input for recent progress by Beaudry and Bobkova-Goerss in $K(2)$-local homotopy theory at the prime 2.
- *Jardine:* gives a sheaf theoretic description of Galois descent and homotopy fixed points problems, with a criterion for descent that uses homotopy types of pro-objects, thereby shedding new light on the étale K-theory spectrum.
- *Kitchloo:* produces a global version of dominant K-theory, thereby extending some of his work with Morava on S^1-equivariant K-theory of the free loop space of a manifold in order to incorporate the action of a compact Lie group.
- *Lawson:* shares welcome insight into his recent proof that the Brown-Peterson spectrum BP at the prime 2 is not an E_∞ ring spectrum, by explaining how he found a certain non-zero Goerss-Hopkins obstruction.
- *Miller:* studies spin formal groups and gives a proof, due to Mike Hopkins, of the Landweber exact functor theorem in all cases of interest, by emphasizing the pivotal role of the height of a formal group.
- *Morava:* contributes to the development of the theory of cyclotomic spectra by considering THH and an analog of the Chern character for rings of integers in certain perfectoid fields.
- *Szymik:* defines chromatic characteristics associated with the Hopkins-Miller classes ζ_n in π_{-1} of the $K(n)$-local sphere, viewing the ζ_n as generalizations of prime numbers, and provides a variety of interesting examples and non-examples.
- *Xu:* details the RP^∞-method for computing differentials in the Adams spectral sequence for the sphere spectrum, providing a simpler example of the technique he recently used with Guozhen Wang to solve a difficult extension problem in the 61-stem.

Acknowledgements: We would like to thank each of the authors and anonymous referees for their time and effort. We also gratefully acknowledge support for the conference from both the National Science Foundation and the host department, especially from its chairman at the time, Matthew Ando.

Daniel G. Davis
Hans-Werner Henn
J.F. Jardine
Mark W. Johnson
Charles Rezk

Plenary Talks

- Agnès Beaudry: *$K(n)$-local Picard Groups and Gross-Hopkins duality*
- Mark Behrens: *The tmf resolution for Z*
- David Blanc: *Higher order homotopy invariants*
- Anna Marie Bohmann: *$CoTHH$ and calculations*
- Hans-Werner Henn: *Resolutions in $K(2)$-local homotopy theory – old and new*
- Kathryn Hess: *Configuration spaces of products*
- Mike Hopkins: *Equivariant dual of Morava E-theory*
- Marc Hoyois: *Motivic infinite loop spaces*
- J.F. Jardine: *Galois cohomological descent*
- Magdalena Kędziorek: *An algebraic model for rational G-spectra*
- Nitu Kitchloo: *Higher Associative structures on Moore Spectra*
- Tyler Lawson: *Higher multiplication and the Brown-Peterson spectrum*
- Jacob Lurie: *Brauer Groups in Stable Homotopy Theory*
- Haynes Miller: *Some homological localization theorems*
- Jack Morava: *Chern characters for Lubin-Tate lifts of $K(n)$*
- Doug Ravenel: *The C_2-equivariant analog of the subalgebra of A generated by Sq^1 and Sq^2*
- Birgit Richter: *Juggling formulae for higher THH*
- Brooke Shipley: *Coalgebras, $coTHH$, and trace maps*
- Zhouli Xu: *Computing stable homotopy groups of spheres*
- Inna Zakharevich: *Constructing derived zeta functions*

Parallel Talks

- Lauren Bandklayder: *The Dold-Thom Theorem via factorization homology*
- Kristine Bauer: *Higher order chain rules and differential structure for abelian functor calculus*
- Haldun Ozgur Bayindir: *Topological equivalences of E_∞ DGAs*
- Georg Biedermann: *A generalized Blakers-Massey Theorem*
- Jeffrey Carlson: *Rational equivariant K-theory of homogeneous spaces*
- Sunil Chebolu: *Strong ghosts in the stable category*
- Martin Frankland: *Eilenberg-MacLane mapping algebras and higher distributivity up to homotopy*
- Jeremy Hahn: *Chromatic types of structured ring spectra*
- Zhen Huan: *Quasi-elliptic cohomology*
- Brenda Johnson: *Functor precalculus*
- Inbar Klang: *Factorization homology and topological Hochschild cohomology of Thom spectra*
- Ben Knudsen: *Subdivisional spaces and graph braid groups*
- Kathryn Lesh: *Tits buildings and fixed points of p-toral groups on decomposition space*
- Ayelet Lindenstrauss: *The topological Hochschild homology of maximal orders in simple algebras over the rationals*
- Carl McTague: tmf *is not a ring spectrum quotient*
- Tasos Moulinos: *On the topological K-theory of derived Azumaya algebras*
- Sam Nariman: *A local to global argument on low dimensional manifolds*
- Peter Nelson: *A small presentation for Morava E-theory power operations*
- Luis Pereira: *Genuine equivariant operads*
- Piotr Pstragowski: *Moduli of Pi-algebras*
- Nima Rasekh: *Representable cartesian fibrations*
- Jonathan Rubin: *On the realization problem for N_∞ operads*
- XiaoLin Danny Shi: *Hurewicz images of Real bordism theory and Morava E-theories*
- Marco Varisco: *Assembly maps for topological cyclic homology*

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This volume contains the proceedings of the conference *Homotopy Theory: Tools and Applications*, in honor of Paul Goerss's 60th birthday, held from July 17–21, 2017, at the University of Illinois at Urbana-Champaign, Urbana, IL.

The articles cover a variety of topics spanning the current research frontier of homotopy theory. This includes articles concerning both computations and the formal theory of chromatic homotopy, different aspects of equivariant homotopy theory and K-theory, as well as articles concerned with structured ring spectra, cyclotomic spectra associated to perfectoid fields, and the theory of higher homotopy operations.

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