CONTEMPORARY MATHEMATICS

740

Centre de Recherches Mathématiques Proceedings

Analytic Methods in Arithmetic Geometry

Arizona Winter School 2016
Analytic Methods in Arithmetic Geometry
March 12–16, 2016
The University of Arizona, Tucson, AZ

Alina Bucur
David Zureick-Brown
Editors

AMS American Mathematical Society

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Preface

This volume contains notes which accompanied the lectures at the nineteenth annual Arizona Winter School, held March 12–16, 2016, at the University of Arizona in Tucson. The Arizona Winter School is an intensive five-day meeting, each year organized around a different central topic in arithmetic geometry, featuring several courses by leading and emerging experts ("an annual pilgrimage," in the words of one participant). The Winter School is the main activity of the Southwest Center for Arithmetic Geometry, which was founded in 1997 by a group of seven mathematicians working in the southwest United States, and which has been supported since that time by the National Science Foundation; in recent years it has been organized in partnership with the Clay Mathematics Institute.

The special character of the Arizona Winter School comes from its format. Each speaker proposes a project, and a month before the Winter School begins, the speaker is assigned a group of graduate students who work on the project. The speakers also provide lecture notes and a bibliography. During the actual school the speaker and and his or her group of students work every evening on the assigned project. On the last day of the workshop, the students from each group present their work to the whole school. The result is a particularly intense and focused five days of mathematical activity (for the students and speakers alike).

The topic of the Winter School in 2016 was Analytic Methods in Arithmetic Geometry, and the speakers were Alina Carmen Cojocaru, Harald Andrés Helfgott, Philippe Michel, and Andrew Sutherland. Étienne Fouvry, Emmanuel Kowalski, and Will Sawin join Michel as co-authors.

We thank the authors for their hard work before, during, and after the Winter School. The anonymous reviewers made numerous valuable comments, and we thank them for their careful reading of this manuscript. We also thank the IAS School of Mathematics and Simons Foundation (#524015) for their support of Alina Bucur, and NSF (grant DMS-1555048) for their support of David Zureick-Brown while editing this volume. Finally, we are indebted to the other members (past and present) of the Southwest Center; it is thanks to their efforts that the Winter School exists in its present form.

Alina Bucur
David Zureick-Brown

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The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

Analytic Methods in Arithmetic Geometry • Bucur and Zureick-Brown, Editors

This volume contains the proceedings of the Arizona Winter School 2016, which was held from March 12–16, 2016, at The University of Arizona, Tucson, AZ.

In the last decade or so, analytic methods have had great success in answering questions in arithmetic geometry and number theory. The School provided a unique opportunity to introduce graduate students to analytic methods in arithmetic geometry.

The book contains four articles. Alina C. Cojocaru's article introduces sieving techniques to study the group structure of points of the reduction of an elliptic curve modulo a rational prime via its division fields. Harald A. Helfgott's article provides an introduction to the study of growth in groups of Lie type, with $\mathrm{SL}_2(\mathbb{F}_q)$ and some of its subgroups as the key examples. The article by Étienne Fouvry, Emmanuel Kowalski, Philippe Michel, and Will Sawin describes how a systematic use of the deep methods from ℓ-adic cohomology pioneered by Grothendieck and Deligne and further developed by Katz and Laumon help make progress on various classical questions from analytic number theory. The last article, by Andrew V. Sutherland, introduces Sato-Tate groups and explores their relationship with Galois representations, motivic L-functions, and Mumford-Tate groups.

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