75 Years of Mathematics of Computation

Symposium
Celebrating 75 Years of Mathematics of Computation
November 1–3, 2018
The Institute for Computational and Experimental Research in Mathematics (ICERM)

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Editors
75 Years of Mathematics of Computation
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Preface

The year 2018 marked the 75th anniversary of the founding of *Mathematics of Computation*, one of the four primary research journals published by the American Mathematical Society and the oldest research journal devoted to computational mathematics. To celebrate this milestone, the symposium “Celebrating 75 Years of Mathematics of Computation” was held during November 1–3, 2018, at the Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (A symposium for the 50th anniversary of the journal took place in Vancouver in 1993.)

The sixteen papers in this volume, written by the symposium speakers and editors of the journal, include both survey articles and new contributions.

On the discrete side, there are four papers covering topics in computational number theory and computational algebra. On the continuous side, there are twelve papers covering topics in machine learning, high dimensional approximations, nonlocal and fractional elliptic problems, gradient flows, hyperbolic conservation laws, Maxwell’s equations, Stokes’s equations, a posteriori error estimation, and iterative methods. Together they provide a snapshot of significant achievements in the past quarter century in computational mathematics and also in important current trends.

We are thankful to the administration and staff members at ICERM for everything they did to make the symposium a great success. Funding for the event by the National Science Foundation is gratefully acknowledged.

Last but not least, we thank the speakers at the symposium and the authors of the papers in this volume for their excellent contributions. A list of the nineteen invited lectures at the symposium in the order that they were presented is provided on the following page.

Susanne C. Brenner
Igor Shparlinski
Chi-Wang Shu
Daniel B. Szyld
Invited presentations at the symposium

- Gunter Malle (Technische Universität Kaiserslautern), Character tables of finite groups.
- Bettina Eick (Technische Universität Braunschweig), The classification of groups of prime-power order.
- Paul Zimmermann (Institut National de Recherche en Informatique et en Automatique), What if Gauss had had a computer?
- Renate Scheidler (University of Calgary), Dan Shanks’ CUFFQI algorithm resurrected.
- Andrew Odlyzko (University of Minnesota), The Riemann Hypothesis and computers.
- Carl Pomerance (Dartmouth College), Primality testing, then and now.
- Chi-Wang Shu (Brown University), Bound-preserving high order schemes for hyperbolic equations - survey and recent developments.
- Wolfgang Dahmen (University of South Carolina), Accuracy controlled computation - the merit of residuals.
- Peter Monk (University of Delaware), Finite element methods for Maxwell’s equations.
- Andrea Bertozzi (University of California, Los Angeles), Graphical models in machine learning, networks and uncertainty quantification.
- Adam M. Oberman (McGill University), Partial differential equations approaches to optimization and regularization of deep neural networks.
- Qiang Du (Columbia University), Nonlocal modeling, analysis and computation.
- Yousef Saad (University of Minnesota), A brief journey to the past of iterative methods for solving sparse linear systems.
- Bjorn Engquist (University of Texas, Austin), Fast solvers for frequency domain wave propagation.
- Ricardo Nochetto (University of Maryland, College Park), Numerical methods for fractional diffusion.
- Olof B. Widlund (New York University), BDDC domain decomposition algorithms.
• Frances Kuo (University of New South Wales), High dimensional integration - the Quasi-Monte Carlo way.

• Jie Shen (Purdue University), A new and robust approach to construct energy stable schemes for gradient flows.

• Douglas Arnold (University of Minnesota), Mathematics of Computation meets geometry.

The full program for the ICERM symposium, including the names and affiliations of all the participants and the slides for most of the invited talks, can be found online at

https://icerm.brown.edu/topical_workshops/tw18-7-ymc/

The slides of a brief presentation of the history of *Mathematics of Computation* from the welcome reception is also available there.
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754 Susanne C. Brenner, Igor Shparlinski, Chi-Wang Shu, and Daniel B. Szyld, Editors, 75 Years of Mathematics of Computation, 2020
752 Samuel Coskey and Grigor Sargsyan, Editors, Trends in Set Theory, 2020
750 A. Bourhim, J. Mashreghi, L. Oubbi, and Z. Abdelali, Editors, Linear and Multilinear Algebra and Function Spaces, 2020
748 Donatella Danielli and Irina Mitrea, Editors, Advances in Harmonic Analysis and Partial Differential Equations, 2020
747 Paul Bruillard, Carlos Ortiz Marrero, and Julia Plavnik, Editors, Topological Phases of Matter and Quantum Computation, 2020
746 Erica Flapan and Helen Wong, Editors, Topology and Geometry of Biopolymers, 2020
745 Federico Binda, Marc Levine, Manh Toan Nguyen, and Oliver Röndigs, Editors, Motivic Homotopy Theory and Refined Enumerative Geometry, 2020
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