CONTEMPORARY MATHEMATICS

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Abelian Varieties and Number Theory

Moshe Jarden
Tony Shaska
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AMERICAN MATHEMATICAL SOCIETY

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Dedicated to Gerhard Frey on his 75th birthday.

Contents

Preface

This volume contains a collection of papers in honor of Gerhard Frey's 75th birthday. Gerhard Frey' s contributions in mathematics over the last five decades have been in number theory, arithmetic and algebraic geometry, field arithmetic, and cryptography. His contribution to Fermat's Last Theorem, by converting it to a problem about elliptic curves is well known. Such elliptic curves today are known as Frey curves. It was Frey's idea and then work of Ribet which connected Fermat's Last Theorem to the Taniyama-Shimura conjecture of elliptic curves which led to the proof of Fermat's Last Theorem by Andrew Wiles. Gerhard Frey also has major contributions to Abelian varieties and their isogenies, field arithmetic, rational points on curves, and many other topics. Gerhard is the single most important person on development of hyperelliptic curve cryptography. He was a director of the Institute of Experimental Mathematics in Essen for many years, editor of Manuscripta Math., and held positions in Erlangen, Saarland, and Essen. He had 19 PhD students until he retired. On the occasion of his 75th birthday we are organizing this volume recognizing his work and achievements. Below we briefly describe each contribution.

Following Gerhard Frey's article [**Fry73**], one calls a field K **PAC** if every nonempty geometrically integral algebraic variety V over K has a K-rational point. It turns out that infinite extensions of finite fields are PAC [**FrJ08**, p. 196, Cor. 11.2.4] (Y. Ershov), non-principal ultra-products of distinct finite fields are PAC, [**FrJ08**, p. 200, Cor. 11.3.4] (J. Ax). In addition, if K is a countable Hilbertian field, then for each positive integer e and for almost all $\sigma := (\sigma_1, \ldots, \sigma_e) \in \mathrm{Gal}(K)^e$ the fixed field $K_{\mathrm{sep}}(\sigma)$ in K_{sep} of $\sigma_1, \ldots, \sigma_e$ is PAC [**FrJ08**, p. 380, Thm. 18.6.1].

Here we denote the algebraic closure of a field K by $\tilde{K}$, the separable algebraic closure of K in $\tilde{K}$ by K_{sep}, the maximal purely inseparable extension of K in $\tilde{K}$ by K_{ins}, and the absolute Galois group $\mathrm{Gal}(K_{\mathrm{sep}}/K)$ of K by $\mathrm{Gal}(K)$. Also, "almost all" is used in the sense of the Haar measure of the profinite group $\mathrm{Gal}(K)^e$.

However, non-separably-closed Henselian fields are not PAC (a result of Frey-Prestel [**FrJ08**, p. 205, Cor. 11.5.5]). In view of this result, Florian Pop has introduced the notion of an ample field that combines both types of fields. A field K is **ample** if for every geometrically integral algebraic variety V over K with a simple K-rational point the set $V(K)$ is Zariski-dense in V (see [**Pop96**, p. 2] or [**Jar11**, p. 68, Def. 5.3.2]). Among others, every PAC field and every Henselian field is ample (a result of Pop, [**Jar11**, Section 5.6]). In addition, every power series field $K_0((X_1, \ldots, X_n))$ over an arbitrary field K_0 is ample [**Pop96**, p. 93, Thm. 5.11.3].

The striking property of an ample field K that Pop proved is that every finite split regular embedding problem over K is solvable. In particular, the inverse

Galois problem over $K(t)$, with t transcendental over K, has a positive solution ([**Pop96**, Main Theorem A] or [**Jar11**, p. 88, Thm. 5.9.2]).

Returning to Frey's contribution to Field Arithmetic, [**FyJ74**] proves for a countable Hilbertian field K, for each $e \geq 1$, for almost all $\sigma \in \mathrm{Gal}(K)^e$, and for every non-zero abelian variety A over $K_{\mathrm{sep}}(\sigma)$, that $\mathrm{rank}(A(K_{\mathrm{sep}}(\sigma)))$ is infinite. This result was generalized for many classes of fields by several authors (e.g. Petersen [**Pet06**] and [**GJ06**]). In particular, Fehm-Petersen proved in [**FeP10**] that

(1) $\mathrm{rank}(A(M)) = \infty$ for every non-zero abelian variety A over an ample field M of characteristic zero.

The proof of (1) relies among others on the Lang-Mordell Conjecture proved by Faltings in [**Fal83**]. In an unpublished paper from 2010, Fehm and Petersen proved (1) also for ample fields K of positive characteristic. The proof relied on a generalization of a theorem of Grauert-Manin:

(2) Let K be a finitely generated extension of $\mathbb{F}_p$ and let C be a geometrically integral algebraic curve over K such that $\mathrm{genus}(C_{\tilde{K}}) \geq 2$ and $C_{\tilde{K}}$ is not birationally equivalent to a curve defined over $\tilde{\mathbb{F}}_p$. Then, $C(K_{\mathrm{ins}})$ is finite.

A proof of (2) was given in [**Kim97**]. However, at that time the proof was unsatisfactory, so Fehm and Petersen did not publish their result.

In the meantime, Damian Rössler has carried out Kim's program and even improved Kim's result by producing a quantitative version of it: Let F be a function field of one variable over an algebraically closed field of positive characteristic p. Let C be a smooth proper geometrically connected curve over F such that $C(F) \neq \emptyset$, $\mathrm{genus}(C_{\tilde{F}}) \geq 2$, and $C_{\tilde{F}}$ is not birationally equivalent to a curve defined over $\tilde{\mathbb{F}}_p$. Let n be the smallest integer with $p^n > 2\,\mathrm{genus}(C_{\tilde{K}}) - 2$ and n is greater than an explicitly given positive integer n_0. Then, $C(F_{\mathrm{ins}})C(F^{p^{-n+1}}) = \emptyset$. Using the result of Grauert-Manin [**Sam66**, p. 107, Thm. 1] about the finiteness of $C(F^{p^{-n+1}})$ (Grauer-Manin), Rössler concludes that $C(F_{\mathrm{ins}})$ is finite.

This has allowed Fehm-Petersen to safely prove (2) (see [**FeP21**, Thm. 3.1]) and to complete the proof of (1) also in positive characteristic under the assumption that M is not an algebraic extension of a finite field [**FeP21**, Thm. 1.1].

Still relating to the main result of [**FyJ74**], Bo-Hae Im and Michael Larsen obsereved that the absolute Galois group $\mathrm{Gal}(K_{\mathrm{sep}}(\sigma))$ is finitely generated (in the sense of profinite groups). That lead them to the following conjecture:

(3) If the absolute Galois group of a field M of characteristic 0 is finitely generated, then $\mathrm{rank}(A(M)) = \infty$ for every non-zero abelian variety A over M. The same holds in positive characteristic provided A is not isotrivial.

Another conjecture, this time of Markus Junker and Jochen Koenigsmann, says that if a field M is infinite and $\mathrm{Gal}(M)$ is finitely generated, then M is ample. It would follows that (3) is a special case of (1).

However, it seems that at this time, nobody has a clue to whether the Junker-Koenigsmann conjecture is true or not. So, in the meantime Im and Larsen have been proving their conjecture in many special cases. Their proofs invoke methods from various branches of Mathematics: Probability, Diophantine Geometry, Combinatorics, Arithmetic, and Analytic Number Theory. The paper [**ImL21**] surveys the special results obtained using those methods.

Akio Tamagawa proves in [**Tam21**] that for every pair (X, Y) of smooth affine curves over $\tilde{\mathbb{F}}_p$ there exists a curve Z admitting étale morphisms to both X and Y. This stands in sharp contrast to a result of Mochizuki saying that there are only finitely many pairs of "general curves" over $\mathbb{C}$ having a common étale covering.

On the other hand, Tamagawa proves that if X is a smooth connected affine curve of genus ≥ 2 over an algebraically closed field F of positive characteristic p and X is not-defined over $\tilde{\mathbb{F}}_p$, then there are only finitely many curves X' as above such that X and X' have a common étale overing. Among others, the proof applies the version of the Mordell-Lang conjecture for algebraically closed fields of positive characteristic proved in [**FeP21**, Prop. 4.2].

Lior Bary-Soroker and Alexei Entin prove the global function field analog of an explicit quantitative version of Hilbert's irreducibility theorem due to S. D. Cohen [**BSE21**]:

If $f(T_1, \ldots, T_n, X)$ is an irreducible polynomial over $\mathbb{F}_q(u)$, then the proportion of n-tuples $(t_1, \ldots, t_n)$ of monic polynomials in u of degree d for which $f(t_1, \ldots, t_n, X)$ is reducible out of all n-tuples of degree d monic polynomials is $O(dq^{-d/2})$. This result appears to fill a forty year old gap in the literature.

Gebhard Böckle and Florian Breuer focus in [**BoB21**] on a Galois representation associated to torsion points of a generic Drinfeld module of arbitrary rank in positive characteristic.

Specifically, the authors consider the ring of integers A of a global function field F of positive characteristic. Let $\mathfrak{p}$ be a maximal ideal of A and let $A_{\mathfrak{p}}$ be the $\mathfrak{p}$-adic completion of A at $\mathfrak{p}$. The group $\mathrm{Gal}(F)$ acts on the universal Tate Module associated to a certain Drinfeld module of rank r. This action gives rise to a continuous representation $\rho\colon \mathrm{Gal}(F) \to \mathrm{GL}_{r-1}(A_{\mathfrak{p}})$. The main result of [**BoB21**] is that ρ is surjective.

Arno Fehm, Dan Haran, and Elad Paran give a new proof to a well known theorem about the realization of every finite group G over $\mathbb{C}(z)$. Their proof uses the method of algebraic patching introduced in [**HV96**] and further presented in [**Jar11**]. In that method one considers the field $\hat{K} = K((t))$ of formal power series in t over a field K, constructs certain finitely many cyclic extensions of $\hat{K}(z)$, for another variable z, and "patches them together" to a G-extension of $\hat{K}(z)$. Having done so, one "pushes down" the latter G-extension to a G-extension of $K(z)$.

In [**FHP21**] the authors carry out the patching procedure directly over $\mathbb{C}(z)$, replacing the locally compact field $\hat{K}$ of the preceding paragraph by the locally compact field $\mathbb{C}$. The proofs of [**FHP21**] assume only knowledge of complex functions theory that an undergraduate student studies.

Anna Cadoret describes in [**Cad21**] an elementary strategy to study the locus where a finite family of linearly independent 1-cohomology classes for the étale fundamental group remains linearly independent under specialization. She applies that strategy to the injectivity of the specialization morphism on the second graded piece of the l-adic Abel-Jacobi filtration on Chow groups varying in the fibers of a smooth projective morphism. When the base scheme is a curve, this provides a generalization of a theorem of Silverman about the sparsity of the jumping locus of the rank in the fibers of an abelian scheme.

David Harbater, Julia Hartman, Valentijn Karemaker, and Florian Pop compare in [**HKP21**] a few local-global principles of smooth varieties over a function field of several variables F over a complete discrete valuation field. They prove that if the local-global principle holds for one family, then it holds for the others. The proof uses, among others, the Artin approximation theorem and resolution of singularities of arithmetic surfaces.

Barry Green and Peter Roquette represent in [**GrR21**] the Deuring theory of reductions of valued function fields of one variable. A function field F/K of one variable is said to have a good reduction with respect to a valuation v of F if there exists a transcendental element $t \in F$ over K such that $[F : K(t)] = [\bar{F} : \bar{K}(\bar{t})]$ where the bar denotes reduction with respect to v and $\bar{t}$ is transcendental over $\bar{K}$ and if, in addition, $\mathrm{genus}(F/K) = \mathrm{genus}(\bar{F}/\bar{K})$. Under this assumption, there is a homomorphism $\mathfrak{a} \to \bar{\mathfrak{a}}$ of the group of divisors of F/K onto the group of divisors of $\bar{F}/\bar{K}$ that maps principal divisors onto the corresponding principal divisors.

The authors give an elementary proof to the statement that an arbitrary function field F/K of one variable has a good reduction at "almost all" valuations of F.

Brendan Creutz and Jossée Phelipe Voloch consider in [**CrV21**] a finite field k, a curve F over k with function field K, and an elliptic curve E over K which is defined over k. It is known from the work of Tate and Milne that the Tate-Shafarevich group $\mathrm{Sha}(E/K)$ is finite, and Milne gave a formula for its order in terms of the Frobenius eigenvalues of E and F.

In this paper the authors use the structure of the set of l-isogenies of elliptic curves to compute the group structure of $\mathrm{Sha}(E/K)$ in the case where E and F are isogenous ordinary elliptic curves.

Yuri G. Zarhin considers in [**Zar21**] a field K of prime characteristic p, an integer $n \geq 5$, an irreducible polynomial $f(x)$ over K of degree n whose Galois group is either the full symmetric group S_n or the alternating group A_n. Let l be an odd prime different from p, $\mathbb{Z}[\zeta_l]$ the ring of integers in the lth cyclotomic field, $C_{f,l} \colon y^l = f(x)$ the corresponding superelliptic curve and $J(C_{f,l})$ its Jacobian. He proves that the ring of all $\tilde{K}$-endomorphisms of $J(C_{f,l})$ coincides with $\mathbb{Z}[\zeta_l]$ if $J(C_{f,l})$ is an ordinary abelian variety and $(l,n) \neq (5,5)$.

Aharon Razon considers in [**Raz21**] a finite dimensional central simple alternative algebra A over a field F of characteristic 0 and let the symmetric group S_n act on the tensor product $A^{\otimes n}$ of n copies of A. He proves that the fixed algebra $\mathrm{Sym}^n A$ under that action is a direct sum of central simple algebras over F. This result generalizes the case where A is associative, which in a sense goes back to Hermann Weyl.

We hope this book will be helpful to all mathematicians working with Abelian varieties, especially those who are crossing over from other areas of mathematics. Our special thanks to all the authors who contributed papers, referees for all their work and effort, and the AMS production staff for their help in preparing this book.

Moshe Jarden

Tony Shaska

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This book is a collection of articles on Abelian varieties and number theory dedicated to Gerhard Frey's 75th birthday. It contains original articles by experts in the area of arithmetic and algebraic geometry.

The articles cover topics on Abelian varieties and finitely generated Galois groups, ranks of Abelian varieties and Mordell-Lang conjecture, Tate-Shafarevich group and isogeny volcanos, endomorphisms of superelliptic Jacobians, obstructions to local-global principles over semi-global fields, Drinfeld modular varieties, representations of etale fundamental groups and specialization of algebraic cycles, Deuring's theory of constant reductions, etc.

The book will be a valuable resource to graduate students and experts working on Abelian varieties and related areas.

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