CONTEMPORARY MATHEMATICS

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Differential Geometry and Global Analysis In Honor of Tadashi Nagano

AMS Special Session Differential Geometry and Global Analysis, Honoring the Memory of Tadashi Nagano (1930-2017) January 16, 2020 Denver, Colorado

> Bang-Yen Chen Nicholas D. Brubaker Takashi Sakai Bogdan D. Suceavă Makiko Sumi Tanaka Hiroshi Tamaru Mihaela B. Vajiac Editors



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This volume represents the celebration of the mathematical legacy of Dr. Tadashi Nagano (January 9, 1930 – February 1, 2017), Professor of Mathematics, University of Tokyo (1959-1967), University of Notre Dame (1967-1986), and Sophia University (1986-2000). Recipient of the Geometry Prize from the Mathematical Society of Japan, 1994.

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Preface

As one of the great Japanese differential geometers, Professor Tadashi Nagano's (1930–2017) career embodied scholarly excellence. He was an inspiring mentor, a dedicated educator, and a creative, one-of-a-kind researcher whose insights about geometry will undoubtedly be felt far into the future. To commemorate his life and work, which impacted the worldwide mathematical community over many decades, the editors of the present volume organized an AMS Special Session at the 2020 Joint Mathematical Meetings (in Denver, Colorado) dedicated to his memory. The celebration took place on January 16, 2020.



FIGURE 1. Professor Makiko Sumi Tanaka at the Joint Mathematical Meetings 2020, in Denver, Colorado, during the AMS Special Session dedicated to the memory of Professor Tadashi Nagano. The picture is taken during a discussion session planned by the organizers, in which the former students and collaborators of T. Nagano were invited to share their memories.

This volume for the *Contemporary Mathematics* series documents the content of that Special Session, exemplifying the mathematical influence of Professor Nagano and providing historical information that is crucial to the development of differential geometry in the second half of the 20th century. Professor Michel L. Lapidus, AMS

PREFACE

Associate Secretary for the Western Section, and Professor Eriko Hironaka, from the AMS editorial team, deserve special recognition for their roles in promoting its completion. The editors also extend the utmost gratitude to the contributors who submitted articles; to the referees, who provided high-quality feedback that immeasurably improved the content and exposition of the material contained in these pages; and to the presenters and participants of the January 16, 2020 celebration of Professor Nagano.

From the inception of this work, the editors planned to invite Professors Bang-Yen Chen, Takushiro Ochiai, and Makiko Sumi Tanaka to write an essay describing Professor Tadashi Nagano's biography and work. This paper opens the present volume. It incorporates testimonials from Professors Richard H. Escobales, Michael Clancy, Jih-Hsin Cheng, and John Burns about their academic work and interactions with Professor Tadashi Nagano.



FIGURE 2. Professor Bang-Yen Chen at the Joint Mathematical Meetings 2020, in Denver, Colorado, during the AMS Special Session dedicated to the memory of Professor Tadashi Nagano. Bang-Yen Chen described his work under the guidance of Tadashi Nagano in his essay *My* education in differential geometry and my indebtness, published in the volume titled Geometry of Submanifolds, No. 756 in the Contemporary Mathematics series.

Additionally, the editors invited contributions from experts who could shed new light on some topics approached in the work of Professor Nagano, including recent developments and generalizations in the geometry of symmetric spaces; minimal surfaces and minimal submanifolds; totally geodesic submanifolds and their classification; Riemannian, affine, projective, and conformal connections; the (M_+, M_-) method and its applications; maximal antipodal subsets; biharmonic and biconservative hypersurfaces in space forms; the geometry of Laplace operator on Riemannian manifolds; Chen-Ricci inequalities for Riemannian maps; and many other topics



FIGURE 3. Professor Hiroshi Tamaru at the Joint Mathematical Meetings 2020, in Denver, Colorado, during the AMS Special Session dedicated to the memory of Professor Tadashi Nagano. Hiroshi Tamaru defended his doctoral dissertation under Tadashi Nagano's supervision at Sophia University in 1998, under the title *The orbit types of symmetric spaces and their applications to* generalized symmetric spaces

that could attract the interest of any scholar working in differential geometry and global analysis on manifolds.

* * *

Tadashi Nagano was born in Taipei in 1930, when Taiwan was administered by Japan. He pursued his undergraduate studies at the University of Tokyo (from 1951 to 1954), and defended his doctoral thesis titled *On compact transformation groups with* (n-1)-*dimensional orbits* under Kentaro Yano's supervision at University of Tokyo in 1959. He worked at the University of Tokyo from April in 1959 to May 1967 as a lecturer (1959–1962) and as an assistant professor (1962–1967). Nagano moved to United States to pursue an academic career with the University of Notre Dame in 1967. Subsequently, he became a full professor of University of Notre Dame in 1969.

Tadashi Nagano was a visiting professor at University of California at Berkeley from 1962–1964, National Tsing Hua University in Taiwan twice, first in 1966 and then one more time in 1978. During his sabbatical leave in 1983, Professor Nagano conducted research at Osaka University. After a successful academic career with



FIGURE 4. From left to right: Tadashi Nagano, Kentaro Yano, and Katsumi Nomizu.

University of Notre Dame, Tadashi Nagano returned to Japan and became a professor with Sophia University in 1986. He retired from Sophia University at 70 years old in 2000.

Most notably, Tadashi Nagano co-authored 10 papers with Shoshichi Kobayashi in the interval 1966–1972, including A theorem on filtered Lie algebras and its applications, Bull. Amer. Math. Soc. 70 (1964), pp. 401–403.

Tadashi Nagano served an editor-in-chief of Tokyo Journal of Mathematics for several years since 1990. In 1994, Tadashi Nagano was presented with the Geometry Prize from Mathematical Society of Japan for his research achievements over a large field of the differential geometry, including a geometric construction of compact symmetric spaces.

The editors express their most profound gratitude for the discussions and the important pieces of historical information provided by Professors Takushiro Ochiai, Koichi Ogiue, and Yusuke Sakane, which helped them prepare the present material. The editors express their highest thanks to Ms. Reiko Nagano and Ms. Junko Nagano for their support and most valuable feedback during their work on this book. PREFACE



FIGURE 5. Professor Tadashi Nagano and his spouse, Mrs. Shizuko Nagano.



FIGURE 6. Tadashi Nagano reading his message of condolence at the Meeting in Memory of Professor Yozo Matsushima, organized by Science School of Osaka University in 1983. Professor Nagano spent his sabbatical leave in 1983 at Osaka University.



FIGURE 7. Professor Richard Escobales at the Joint Mathematical Meetings 2020, in Denver, Colorado, during the AMS Special Session dedicated to the memory of Professor Tadashi Nagano.

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This volume contains the proceedings of the AMS Special Session on Differential Geometry and Global Analysis, Honoring the Memory of Tadashi Nagano (1930–2017), held January 16, 2020, in Denver, Colorado.

Tadashi Nagano was one of the great Japanese differential geometers, whose fundamental and seminal work still attracts much interest today. This volume is inspired by his work and his legacy and, while recalling historical results, presents recent developments in the geometry of symmetric spaces as well as generalizations of symmetric spaces; minimal surfaces and minimal submanifolds; totally geodesic submanifolds and their classification; Riemannian, affine, projective, and conformal connections; the (M_+, M_-) method and its applications; and maximal antipodal subsets. Additionally, the volume features recent achievements related to biharmonic and biconservative hypersurfaces in space forms, the geometry of Laplace operator on Riemannian manifolds, and Chen-Ricci inequalities for Riemannian maps, among other topics that could attract the interest of any scholar working in differential geometry and global analysis on manifolds.





