

CONTEMPORARY MATHEMATICS

782

Recent Trends in Formal and Analytic Solutions of Diff. Equations

Virtual Conference
Formal and Analytic Solutions of Diff. Equations
June 28–July 2, 2021
University of Alcalá, Alcalá de Henares, Spain

Galina Filipuk
Alberto Lastra
Sławomir Michalik
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Preface

The book presents the state of the art in the study of functional equations in the complex domain, including differential equations, difference equations, q -difference equations, moment-differential equations, partial differential equations, etc. For this reason, the title of the volume contains the word “diff.” aiming to embrace all of them together. The volume consists of selected peer-reviewed contributions from the conference “Formal and Analytic Solutions of Diff. Equations on the Internet”, which was held online from June 28th to July 2nd, 2021. The editors want to express sincere gratitude to all the participants of the conference who made it possible to exchange scientific ideas and results, to examine different directions for future research and to promote both new and existing collaborations of researchers in these topics, despite the world pandemic situation. The virtual conference venue was the School of Architecture of the University of Alcalá, located in the city center of Alcalá de Henares (Spain).

The topics of the conference were:

- Ordinary differential equations in the complex domain. Formal and analytic solutions. Stokes multipliers.
- Formal and analytic solutions of partial differential equations.
- Formal and analytic solutions of difference equations (including q -difference and differential-difference equations).
- Special functions (hypergeometric functions and others), orthogonal polynomials, continuous and discrete Painlevé equations.
- Integrable systems.
- Holomorphic vector fields. Normal forms.
- Asymptotic expansions, Borel summability.

Scientific Committee consisted of Galina Filipuk (University of Warsaw, Poland), Alberto Lastra (University of Alcalá, Spain), Stephane Malek (University of Lille, France), Jorge Mozo-Fernández (University of Valladolid, Spain), Yasunori Okada (Chiba University, Japan), Hidetoshi Tahara (Sophia University, Japan).

Organizing Committee consisted of Javier Jiménez-Garrido (University of Cantabria, Spain), Alberto Lastra (University of Alcalá, Spain), Ignacio Miguel (University of Valladolid, Spain), Andrew Pickering (Universidad Rey Juan Carlos, Spain).

This volume presents recent advances in the study of functional equations made by experts in the field. A. D. Bruno considers normal forms of a polynomial ordinary differential equation (ODE) and A. B. Batkhin studies the procedure of deriving homological equations of arbitrary order for the Hamiltonian normal form. Hamiltonian systems are also considered from the summability point of view in the contribution by M. Yoshino. E. Ciechanowicz deals with the so-called Painlevé sigma equations from the Nevanlinna theory point of view. G. Filipuk, A. Ligęza

and A. Stokes focus on the third Painlevé equation and present relations between different Hamiltonian forms of this equation. In the last years, one finds a growing interest in the study of q -equations from the analytic and formal point of view. The contribution by S. Sasaki, S. Takagi and K. Takemura is an example of this direction. They consider the so-called q -Heun equation. The contribution by C. Zhang concerns the positive powers of the q -analogs of Euler series, which is of great interest and importance in the theory of summability of formal solutions to functional equations. The contribution by H. Ogawara deals with the differential transcendence of solutions for q -difference equation of Ramanujan function. Systems of difference equations coming from orthogonal polynomials are studied in the work by Y. Chen, G. Filipuk and M. N. Rebocho. Singular perturbed equations are also of interest in this volume. The work by T. Aoki and S. Uchida is an example of this. They search for the structure of the Voros coefficients of the generalized hypergeometric differential equation. T. Oshima studies Riemann-Liouville transform of linear differential equations on the Riemann sphere. M. Cafasso and S. Tarricone apply the Riemann-Hilbert problem technique to the Airy process. H. Tahara considers a family of nonlinear singular second order partial differential equations in the complex plane, determines uniqueness of solutions, and applies this result to the analytic continuation of solutions. As mentioned above, the theory of summability of formal solutions to functional equations is also considered in this volume. We refer to the contribution by M. Suwińska on the study of moment functional equations, and more precisely on moment integro-differential equations. The volume concludes with the state of the art regarding such emerging functional equations, in a work by A. Lastra, S. Michalik and M. Suwińska.

We hope that the present volume will be of interest not only for researchers in this field of knowledge but also for graduate students who aim to acquire new techniques and learn new results.

G. Filipuk
A. Lastra
S. Michalik

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This volume contains the proceedings of the conference on Formal and Analytic Solutions of Diff. Equations, held from June 28–July 2, 2021, and hosted by University of Alcalá, Alcalá de Henares, Spain.

The manuscripts cover recent advances in the study of formal and analytic solutions of different kinds of equations such as ordinary differential equations, difference equations, q -difference equations, partial differential equations, moment differential equations, etc. Also discussed are related topics such as summability of formal solutions and the asymptotic study of their solutions.

The volume is intended not only for researchers in this field of knowledge but also for students who aim to acquire new techniques and learn recent results.



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