

CONTEMPORARY MATHEMATICS

784

Advances in Inverse Problems for Partial Differential Equations

AMS Special Session
Recent Developments on Analysis and Computation for
Inverse Problems for PDEs
March 13–14, 2021, Virtual

AMS Special Session
Recent Advances in Inverse Problems for PDEs
October 23–24, 2021, Virtual

Dinh-Liem Nguyen
Loc Hoang Nguyen
Thi-Phong Nguyen
Editors

Advances in
Inverse Problems for
Partial Differential Equations

CONTEMPORARY MATHEMATICS

784

Advances in Inverse Problems for Partial Differential Equations

AMS Special Session
Recent Developments on Analysis and Computation for
Inverse Problems for PDEs
March 13–14, 2021, Virtual

AMS Special Session
Recent Advances in Inverse Problems for PDEs
October 23–24, 2021, Virtual

Dinh-Liem Nguyen
Loc Hoang Nguyen
Thi-Phong Nguyen
Editors

EDITORIAL COMMITTEE

Michael Loss, Managing Editor

John Etnyre Angela Gibney Catherine Yan

2020 *Mathematics Subject Classification*. Primary 35J05, 35R30, 49N45, 65M32, 65N21, 65R20, 78A46, 78M22, 80A23, 86A22.

Library of Congress Cataloging-in-Publication Data

Names: Nguyen, Dinh-Liem, 1986- editor.

Title: Advances in inverse problems for partial differential equations : virtual AMS special session on recent developments on analysis and computation for inverse problems for PDEs, March 13-14, 2021; virtual AMS sectional meeting on recent advances in inverse problems for PDEs, October 23-24, 2021 / Dinh-Liem Nguyen, Loc Hoang Nguyen, Thi-Phong Nguyen, editors.

Description: Providence, Rhode Island : American Mathematical Society, [2023] | Contemporary mathematics, 0271-4132 ; volume 784 | Includes bibliographical references.

Identifiers: LCCN 2022046723 | ISBN 9781470469689 (paperback)

Subjects: Partial differential equations – Elliptic equations and systems – Laplacian operator, reduced wave equation (Helmholtz equation), Poisson equation. | Partial differential equations – Miscellaneous topics – Inverse problems. | Calculus of variations and optimal control; optimization – Miscellaneous topics – Inverse problems. | Numerical analysis – Partial differential equations, initial value and time-dependent initial-boundary value problems – Inverse problems. | Numerical analysis – Partial differential equations, boundary value problems – Inverse problems. | Numerical analysis – Integral equations, integral transforms – Integral equations. | Optics, electromagnetic theory – General – Inverse scattering problems. | Optics, electromagnetic theory – Basic methods – Spectral methods. | Classical thermodynamics, heat transfer – Thermodynamics and heat transfer – Inverse problems. | Geophysics – Geophysics – Inverse problems.

Classification: LCC QA374 .A24 2023

LC record available at <https://lcn.loc.gov/2022046723>

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: <https://doi.org/10.1090/conm/784>

Color graphic policy. Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Publisher. In general, color graphics will appear in color in the online version.

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit www.ams.org/publications/pubpermissions.

Send requests for translation rights and licensed reprints to reprint-permission@ams.org.

© 2023 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <https://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 28 27 26 25 24 23

Contents

Preface	vii
Discretization and convergence of the EIT optimal control problem in Sobolev spaces with dominating mixed smoothness UGUR G. ABDULLA and SALEHEH SEIF	1
Global reconstruction of initial conditions of nonlinear parabolic equations via the Carleman-contraction method THUY T. LE	23
Regularization of the factorization method with applications to inverse scattering ISAAC HARRIS	43
Sampling type method combined with deep learning for inverse scattering with one incident wave THU LE, DINH-LIEM NGUYEN, VU NGUYEN, and TRUNG TRUONG	63
Fast numerical solutions to direct and inverse scattering for bi-anisotropic periodic Maxwell's equations DINH-LIEM NGUYEN and TRUNG TRUONG	81
Reconstructing a space-dependent source term via the quasi-reversibility method LOC H. NGUYEN and HUONG T.T. VU	103
Convergence analysis of Nédélec finite element approximations for a stationary Maxwell's system QUYEN TRAN	119
Quasi-reversibility method and neural network machine learning for forecasting of stock option prices MIKHAIL V. KLIBANOV, KIRILL V. GOLUBNICHY, and ANDREY V. NIKITIN	129
Numerical reconstruction for 3D nonlinear SAR imaging via a version of the convexification method VO ANH KHOA, MICHAEL VICTOR KLIBANOV, WILLIAM GRAYSON POWELL, and LOC HOANG NGUYEN	145
Initial state reconstruction on graphs VO ANH KHOA, MAI THANH NHAT TRUONG, IMHOTEP HOGAN, and ROSELYN WILLIAMS	169

Active control of scalar Helmholtz fields in the presence of known impenetrable obstacles

LANDER BESABE and DANIEL ONOFREI

189

Preface

An inverse problem for a partial differential equation (PDE) typically aims to determine some information about the PDE such as a coefficient, an initial condition, and geometry of a domain, from some knowledge about the solution of the PDE. Inverse problems for PDEs arise in a variety of applications including non-destructive testing, radar, medical imaging, geophysical exploration, image processing, finance, etc. These inverse problems are ill-posed problems where the solution usually does not depend continuously on the measured data. Also, in many situations, the dependence of the solution on the data is nonlinear. These features cause substantial challenges in computing numerical solutions to inverse problems for PDEs. During the past three decades, this area of research has been one of the fastest-growing areas in applied mathematics with significant developments in both numerical methods and theoretical analysis.

This volume focuses on new developments in numerical methods for solving inverse problems for PDEs. It contains eleven papers contributed by speakers of two special sessions of the American Mathematical Society Sectional Meeting: “Recent developments on Analysis and Computation for Inverse Problems for PDEs”, virtually held from March 13-14, 2021, and “Recent Advances in Inverse Problems for Partial Differential Equations”, virtually held from October 23-24, 2021. These papers are concerned with various research topics such as electrical impedance tomography, inverse scattering in radar and optics, reconstruction of initial conditions, control of acoustic fields, and stock price forecasting. The authors studied iterative and non-iterative approaches such as optimization-based, globally convergent, sampling, and machine learning-based methods.

We truly thank all speakers in the special sections and the authors of the papers in this volume for their excellent contributions. We are especially thankful to the administration and staff members of the American Mathematical Society for everything they did to guarantee the success of the meetings and this volume. We hope you enjoy this volume.

Dinh-Liem Nguyen
Loc Hoang Nguyen
Thi-Phong Nguyen

Selected Published Titles in This Series

- 784 **Dinh-Liem Nguyen, Loc Hoang Nguyen, and Thi-Phong Nguyen, Editors,** *Advances in Inverse Problems for Partial Differential Equations*, 2023
- 781 **Emanuel Indrei, Diego Marcon, and Levon Nurbekyan, Editors,** *Geometric and Functional Inequalities and Recent Topics in Nonlinear PDEs*, 2023
- 780 **Erik Koelink, Stefan Kolb, Nicolai Reshetikhin, and Bart Vlaar, Editors,** *Hypergeometry, Integrability and Lie Theory*, 2022
- 779 **Samuele Anni, Valentijn Karemaker, and Elisa Lorenzo García, Editors,** *Arithmetic, Geometry, Cryptography, and Coding Theory 2021*, 2022
- 778 **Carlos Galindo, Alejandro Melle Hernández, Julio José Moyano-Fernández, and Wilson A. Zúñiga-Galindo, Editors,** *p -Adic Analysis, Arithmetic and Singularities*, 2022
- 777 **Bang-Yen Chen, Nicholas D. Brubaker, Takashi Sakai, Bogdan D. Suceavă, Makiko Sumi Tanaka, Hiroshi Tamaru, and Mihaela B. Vajiac, Editors,** *Differential Geometry and Global Analysis*, 2022
- 776 **Aaron Wootton, S. Allen Broughton, and Jennifer Paulhus, Editors,** *Automorphisms of Riemann Surfaces, Subgroups of Mapping Class Groups and Related Topics*, 2022
- 775 **Fernando Galaz-García, Cecilia González-Tokman, and Juan Carlos Pardo Millán, Editors,** *Mexican Mathematicians in the World*, 2021
- 774 **Randall J. Swift, Alan Krinik, Jennifer M. Switkes, and Jason H. Park, Editors,** *Stochastic Processes and Functional Analysis*, 2021
- 773 **Nicholas R. Baeth, Thiago H. Freitas, Graham J. Leuschke, and Victor H. Jorge Pérez, Editors,** *Commutative Algebra*, 2021
- 772 **Anatoly M. Vershik, Victor M. Buchstaber, and Andrey V. Malyutin, Editors,** *Topology, Geometry, and Dynamics*, 2021
- 771 **Nicolás Andruskiewitsch, Gongxiang Liu, Susan Montgomery, and Yinhua Zhang, Editors,** *Hopf Algebras, Tensor Categories and Related Topics*, 2021
- 770 **Stéphane Ballet, Gaetan Bisson, and Irene Bouw, Editors,** *Arithmetic, Geometry, Cryptography and Coding Theory*, 2021
- 769 **Kiyoshi Igusa, Alex Martsinkovsky, and Gordana Todorov, Editors,** *Representations of Algebras, Geometry and Physics*, 2021
- 768 **Dražen Adamović, Andrej Dujella, Antun Milas, and Pavle Pandžić, Editors,** *Lie Groups, Number Theory, and Vertex Algebras*, 2021
- 767 **Moshe Jarden and Tony Shaska, Editors,** *Abelian Varieties and Number Theory*, 2021
- 766 **Paola Comparin, Eduardo Esteves, Herbert Lange, Sebastián Reyes-Carocca, and Rubí E. Rodríguez, Editors,** *Geometry at the Frontier*, 2021
- 765 **Michael Aschbacher,** *Quaternion Fusion Packets*, 2021
- 764 **Gabriel Cunningham, Mark Mixer, and Egon Schulte, Editors,** *Polytopes and Discrete Geometry*, 2021
- 763 **Tyler J. Jarvis and Nathan Priddis, Editors,** *Singularities, Mirror Symmetry, and the Gauged Linear Sigma Model*, 2021
- 762 **Atsushi Ichino and Kartik Prasanna,** *Periods of Quaternionic Shimura Varieties. I.*, 2021
- 761 **Ibrahim Assem, Christof Geiß, and Sonia Trepode, Editors,** *Advances in Representation Theory of Algebras*, 2021
- 760 **Olivier Collin, Stefan Friedl, Cameron Gordon, Stephan Tillmann, and Liam Watson, Editors,** *Characters in Low-Dimensional Topology*, 2020

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/conmseries/.

This volume contains the proceedings of two AMS Special Sessions “Recent Developments on Analysis and Computation for Inverse Problems for PDEs,” virtually held on March 13-14, 2021, and “Recent Advances in Inverse Problems for Partial Differential Equations,” virtually held on October 23-24, 2021.

The papers in this volume focus on new results on numerical methods for various inverse problems arising in electrical impedance tomography, inverse scattering in radar and optics problems, reconstruction of initial conditions, control of acoustic fields, and stock price forecasting. The authors studied iterative and non-iterative approaches such as optimization-based, globally convergent, sampling, and machine learning-based methods.

The volume provides an interesting source on advances in computational inverse problems for partial differential equations.



ISBN 978-1-4704-6968-9



9 781470 469689

CONM/784