CONTEMPORARY MATHEMATICS

784

# Advances in Inverse Problems for Partial Differential Equations

AMS Special Session
Recent Developments on Analysis and Computation for Inverse Problems for PDEs
March 13–14, 2021, Virtual

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October 23–24, 2021, Virtual

Dinh-Liem Nguyen
Loc Hoang Nguyen
Thi-Phong Nguyen
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# Preface

An inverse problem for a partial differential equation (PDE) typically aims to determine some information about the PDE such as a coefficient, an initial condition, and geometry of a domain, from some knowledge about the solution of the PDE. Inverse problems for PDEs arise in a variety of applications including non-destructive testing, radar, medical imaging, geophysical exploration, image processing, finance, etc. These inverse problems are ill-posed problems where the solution usually does not depend continuously on the measured data. Also, in many situations, the dependence of the solution on the data is nonlinear. These features cause substantial challenges in computing numerical solutions to inverse problems for PDEs. During the past three decades, this area of research has been one of the fastest-growing areas in applied mathematics with significant developments in both numerical methods and theoretical analysis.

This volume focuses on new developments in numerical methods for solving inverse problems for PDEs. It contains eleven papers contributed by speakers of two special sessions of the American Mathematical Society Sectional Meeting: "Recent developments on Analysis and Computation for Inverse Problems for PDEs", virtually held from March 13-14, 2021, and "Recent Advances in Inverse Problems for Partial Differential Equations", virtually held from October 23-24, 2021. These papers are concerned with various research topics such as electrical impedance tomography, inverse scattering in radar and optics, reconstruction of initial conditions, control of acoustic fields, and stock price forecasting. The authors studied iterative and non-iterative approaches such as optimization-based, globally convergent, sampling, and machine learning-based methods.

We truly thank all speakers in the special sections and the authors of the papers in this volume for their excellent contributions. We are especially thankful to the administration and staff members of the American Mathematical Society for everything they did to guarantee the success of the meetings and this volume. We hope you enjoy this volume.

Dinh-Liem Nguyen
Loc Hoang Nguyen
Thi-Phong Nguyen

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The papers in this volume focus on new results on numerical methods for various inverse problems arising in electrical impedance tomography, inverse scattering in radar and optics problems, reconstruction of initial conditions, control of acoustic fields, and stock price forecasting. The authors studied iterative and non-iterative approaches such as optimization-based, globally convergent, sampling, and machine learning-based methods.

The volume provides an interesting source on advances in computational inverse problems for partial differential equations.

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