

# CONTEMPORARY MATHEMATICS

785

## Algebra and Coding Theory

Virtual Conference in Honor of Tariq Rizvi  
Noncommutative Rings and their Applications VII

July 5–7, 2021

Université d'Artois, Lens, France

Virtual Conference

Quadratic Forms, Rings and Codes

July 8, 2021

Université d'Artois, Lens, France

A. Leroy

S. K. Jain

Editors

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This volume is dedicated to Syed Tariq Rizvi for his outstanding contributions to Mathematics and in particular to Rings and Modules.



Courtesy of Professor André Leroy

*Tariq Rizvi, Ohio State University, Lima, Ohio.*



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## Preface

As an outgrowth of the two conferences "Noncommutative rings and their Application (NCRA)" and "Quadratic forms, rings and codes(QFRC)" held online in July 2021 in Lens (France), this volume contains a variety of topics in algebra and exhibits their interplay as presented in the meetings by the participants.

The NCRA conference was dedicated to our colleague Tariq Rizvi. Tariq Rizvi is a prolific algebraist and his impact on rings and modules is tremendous. His influence is really impressive and many algebraists around the world benefited from his experience, expertise and generosity. He is frank, humble, generous and it is always a pleasure to meet him with his welcoming smile. We are lucky and proud to count him as our friend and we are very happy to give him, through this volume, a small token of our respect and gratitude. Some of the papers included in the volume reflect the influence of his work on the questions related to the direct sums of continuous and quasi-continuous modules, among others.

One of the aims of these meetings was to establish connections between ring theory and coding theory. In particular, the applications to coding theory of Frobenius rings, the skew group rings and iterated Ore extensions are the subject of some of the papers. The questions on the classical topics, such as Utumi rings, Baer rings, nil and nilpotent algebras, Brauer groups are discussed in some papers. Other articles deal with questions that relate to the elementwise study for rings and modules. Lastly, this volume includes papers dealing with questions in homological algebra and lattice theory. All these works show the vivacity of the research in the area of noncommutative rings and its influence on other subjects.

As usual, the papers went through a strict refereeing process and we would like to use this opportunity to thank all the anonymous referees who kindly reviewed the papers.

We thank Christine Thivierge who was so helpful and efficient in handling this publication.

André Leroy and S.K. Jain



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The articles cover topics in commutative and noncommutative algebra and applications to coding theory. In some papers, applications of Frobenius rings, the skew group rings, and iterated Ore extensions to coding theory are discussed. Other papers discuss classical topics, such as Utumi rings, Baer rings, nil and nilpotent algebras, and Brauer groups. Still other articles are devoted to various aspects of the elementwise study for rings and modules. Lastly, this volume includes papers dealing with questions in homological algebra and lattice theory. The articles in this volume show the vivacity of the research of noncommutative rings and its influence on other subjects.



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