CONTEMPORARY MATHEMATICS

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Mathematical and Computational Modeling of Phenomena Arising in Population Biology and Nonlinear Oscillations

Abba B. Gumel
Editor

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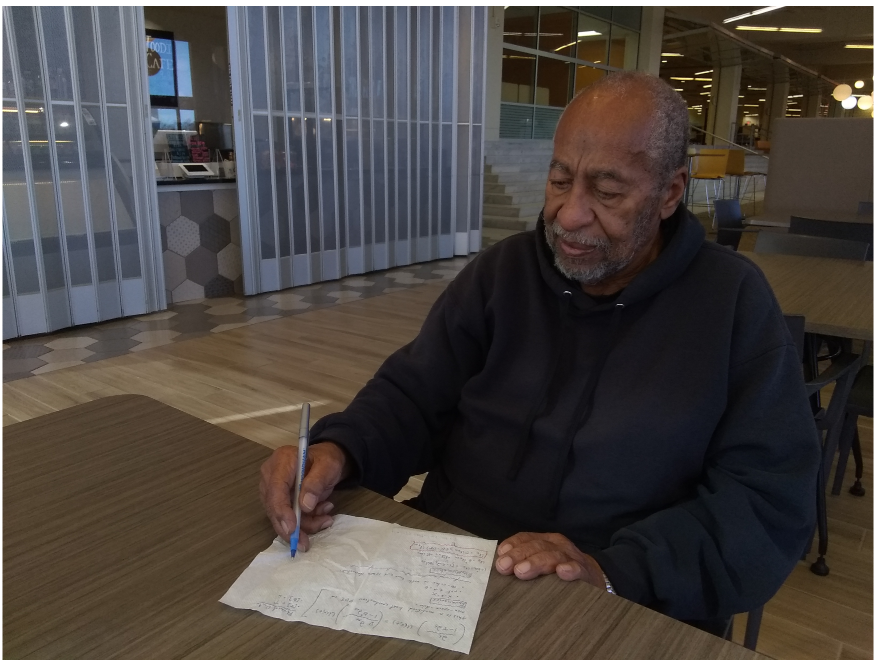
To Professor Ronald E. Mickens on the occasion of his 80th birthday



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Preface

This book contains a series of chapters, contributed in honor of the 80th birthday of Professor Ronald E. Mickens (Distinguished Callaway Professor of Physics Emeritus, Clark Atlanta University, Georgia, USA), that address some of the current challenges and advances on the study of the mathematical and computational modeling of real-life phenomena arising in population biology and nonlinear oscillation. Professor Mickens is renowned for his stellar contributions in nonlinear oscillations and computational mathematics, particularly the design and analysis of robust and dynamically-consistent *nonstandard finite difference* methods for discretizing continuous-time dynamical systems arising from the mathematical modeling of phenomena in the natural and engineering sciences. The papers contained in this volume are contributed by collaborators of, and researchers whose work was inspired by the works of, Professor Mickens. Some of the contributors spoke at the AMS Special Session on Advances in the Applications of Nonstandard Finite Difference Methods, January 6-9, 2021, Atlanta, Georgia.

The volume contains papers in three main areas, notably the design, analysis and applications of nonstandard finite difference discretization of continuous-time dynamical systems in the natural and engineering sciences, mathematical population biology (with emphasis on the design, analysis, parameterizations of mathematical models for gaining insight into the transmission dynamics and control of emerging and re-emerging infectious diseases, such as COVID-19 and malaria), and nonlinear oscillations. The volume is a collection of 16 papers, containing some new results and techniques. In particular, the papers highlight numerous new advances and challenges associated with the design of dynamically-consistent discrete-time models for approximating the solutions of differential equations (both ordinary and partial), based on Mickens's nonstandard finite-difference discretization framework. Some new higher-order techniques for designing nonstandard finite-difference methods for phenomena in the biological and chemical processes, as well as a generalized autonomous dynamical system, are proposed. Furthermore, the volume contains papers that focus on the design and analysis of robust models for gaining insight and understanding on the spread and mitigation of infectious diseases. In particular, the book contains chapters on modeling the spread and control of the 2019 novel Coronavirus pandemic (COVID-19, which represents the greatest public health problem human have faced since the 1918/1919 influenza pandemic), on the development of a novel genetic-epidemiology modeling framework for malaria disease (the most important parasitic disease of mankind) and a discrete-time epidemic model with Ricker growth. The book also contains papers on mathematical ecology, including works on the effects of temperature and prescribed fire on the dynamics of tick-borne diseases, discretization of models for microbial population with environmental stress, Monod kinetics with half saturation in a chemostat setting etc. Each of the papers is peer-reviewed by at least two anonymous reviewers.

The editor acknowledges the support of the American Mathematical Society. In particular, special thanks to Ms. Christine M. Thivierge (Editorial Assistant, AMS) and the Production and Technical Staff for the great work throughout this process. I am grateful for their careful reading, diligence and efficiency. I am immensely thankful to the authors who contributed their excellent chapters and to the many anonymous reviewers for their constructive and diligent reviews.

Abba B. Gumel

May 2023

Career of Professor Ronald E. Mickens

Professor Mickens graduated summa cum laude from Fisk University in 1964 with a B.A. degree in Physics and Mathematics. His graduation honors included election to Phi Beta Kappa. Doctoral studies in theoretical Physics were completed in 1968 at Vanderbilt University. For the next two years, as a National Science Foundation Postdoctoral Fellow at the Center for Theoretical Physics, the Massachusetts Institute of Technology, he did research on calculating and studying the scattering amplitudes for strongly interacting elementary particles at high energies. In 1970, he returned to Fisk University to teach and do research in Mathematical Physics.

Professor Mickens' major research activities from 1966-1977 were centered on the application of scattering theory to problems in both elementary particle physics and chemical reaction rates. He showed that the chemical equilibrium coefficient could be directly constructed from the general principles of scattering theory. He also applied these techniques to obtain new results for the low temperature behavior of recombination reaction rate functions. In addition, Mickens showed how to generalize the fundamental Pomeranchuk Theorem, which relates various physical properties of particle and anti-particle collisions at very high energies, to complex processes having two-and many-particle final states.

During 1977-1981, Professor Mickens began a study of second-order, ordinary differential equations which model nonlinear oscillatory dynamical systems. One of the results from this research was the publication of the book, Nonlinear Oscillations (Cambridge University Press, New York, 1981). Subsequent work in this area led to more than 50 publications related to the analysis and construction of uniformly valid perturbation and global methods to calculate analytical approximations to the solutions of such equations. A major tool used for these investigations was the rewriting of the original second-order ODEs to system form and then studying the behavior of the trajectories in the resulting two-dimensional phase-space.

In 1981, Professor Mickens was a Visiting Research Fellow at the Joint Institute for Laboratory Astrophysics located in Boulder, CO. There he began a study of difference equations and the instabilities that can arise in the numerical integrations of differential equations by the use of finite-difference methods. This work was the genesis of a research program, which allowed the construction of what is now called "nonstandard finite-difference schemes." Discrete models based on these techniques provide superior numerical solutions, as compared with many standard methods, and allow the direct incorporation of important physical properties of the dynamical systems such as positivity and boundedness. The general philosophy of these techniques, as well as their application to problems in the natural and engineering sciences are given in two books: R.E. Mickens: Finite Difference Models of Differential Equations (World Scientific, Singapore, 1994) and R.E. Mickens (editor): Applications of Nonstandard Finite Difference Schemes (World Scientific, Singapore, 2000).

One of the highlights of Professor Mickens' career was the publication of a book that provided a broad introduction to finite-difference equations. This book, R.E. Mickens: Difference Equations (Van Nostrand Reinhold, New York, 1987) was one of the first modern textbooks on the subject. The demand for this textbook, coupled with a general public interest in difference equations, iteration procedures and discrete modeling, led to a second greatly expanded edition, R.E. Mickens: Difference Equations, Theory and Applications (Van Nostrand Reinhold, New York, 1990). A significant feature of the latter book is its final chapter. In it, various difference equations arising from the modeling of phenomena in physics, chemistry, the biosciences, numerical integration and asymptotics are presented and discussed, along with references to the appropriate research literature. A third edition was published in 2015.

Throughout his career, Professor Mickens has received numerous fellowships, some of which are: Woodrow Wilson Fellowship (1964-1965); Danforth Foundation Fellowship (1965-1968); National Science Foundation Postdoctoral Fellowship (1968-1970); and joint Institute for Laboratory Astrophysics Visiting Research Fellowship (1981-1982). His awards include: selection as the Distinguished Fuller E. Callaway Professor at Atlanta University (now Clark Atlanta University) in 1985 to the present; the National Association of Mathematicians Distinguished Service Award (1996); election to Fellowship in the American Physical Society (1999); and service for a two-year term (2000-2002) as a Distinguished Lecturer of Sigma Xi, the Scientific Research Society of America. In 2018, Mickens was the recipient of the Blackwell-Tapia Prize in Mathematics. Since 1970, his research efforts have been continuously funded by grants from the Army Research Office, Department of Energy, NASA, National Institutes of Health, and National Science Foundation.

Professor Mickens has made more than 600 invited presentations at national and international scientific/mathematical conferences and at various colleges and universities. He has served as a member of the editorial board of several research journals; these include the Journal of Difference Equations and Applications, Journal of Sound and Vibration, and Journal on Applied Sciences and Computation. He has published more than 350 refereed research articles, written 12 books and edited seven volumes. Of interest to those with a philosophical leaning is the following volume that deals with the relationship between science and mathematics, R.E. Mickens (editor): Mathematics and Science (World Scientific, Singapore, 1990).

In the early 2000's, Professor Mickens extended the concept of "dynamic consistency" and used it to place limitations on the structure of mathematical models for physical systems and their discretizations. He demonstrated that models may not satisfy all the relevant fundamental laws and other requirements of physics, yet still be useful for predicting the properties of particular aspects of a system. He also has investigated periodic solutions associated with simple, closed, convex curves in the plane, with the origin an interior point of the curve, and showed that three periodic functions could be generated. This work is summarized in the book, R.E. Mickens: Generalized Trigonometric and Hyperbolic Functions (CRC Press, 2019).

Finally, Professor Mickens' current research involves (a) the construction of approximate and exact solutions to both linear and nonlinear difference and differential equations; (b) the study of the qualitative properties of solutions to strongly nonlinear, oscillatory, second-order differential equations; (c) the continuation of work on nonstandard finite difference schemes for the numerical integration of differential equations; and, (d) the gathering of relevant materials and the publication of scholarly articles on the history and sociology of science as it relates to African Americans. All of these exciting efforts are supported by a growing list of national and international collaborators who provide not only valued scientific input and stimulating questions/comments/critiques, but who are also both good colleagues and friends.

Selective List of Major Oral Interviews

(1) American Institute of Physics (2020 August 5 -7, 10, 11, and 13). Web Site:
`https://www.aip.org/history-programs/niels-bohr-library/oral-histories/47213`

(2) Atomic Heritage Foundation (July 30, 2018). Web Site:
`https://ahf.nuclearmuseum.org/voices/oral-histories/ronald-e-mickens-interview/`

(3) The HistoryMakers (December 11, 2006). Web Site:
`https://www.thehistorymakers.org/biography/ronald-mickens-41`

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Mathematics of Population Dynamics and Nonlinear Oscillations • Gumel, Editor

This book consists of a series of papers focusing on the mathematical and computational modeling and analysis of some real-life phenomena in the natural and engineering sciences. The book emphasizes three main themes: (i) the design and analysis of robust and dynamically-consistent nonstandard finite-difference methods for discretizing continuous-time dynamical systems arising in the natural and engineering sciences, (ii) the mathematical study of nonlinear oscillations, and (iii) the design and analysis of models for the spread and control of emerging and re-emerging infectious diseases.

Specifically, some of the topics covered in the book include advances and challenges on the design, analysis and implementation of nonstandard finite-difference methods for approximating the solutions of continuous-time dynamical systems, the design and analysis of models for the spread and control of the COVID-19 pandemic, modeling the effect of prescribed fire and temperature on the dynamics of tick-borne disease, and the design of a novel genetic-epidemiology framework for malaria transmission dynamics and control.

The book also covers the impact of environmental factors on diseases and microbial populations, Monod kinetics in a chemostat setting, structure and evolution of poroacoustic solitary waves, mathematics of special (periodic) functions and the numerical discretization of a phase-lagging equation with heat source.

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