

CONTEMPORARY MATHEMATICS

794

Recent Advances in Diffeologies and Their Applications

AMS-EMS-SMF Special Session
Recent Advances in Diffeologies and Their Applications
July 18–20, 2022
Université de Grenoble-Alpes, Grenoble, France

Jean-Pierre Magnot
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In memory of Jean-Marie Souriau

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Preface

In the current state of knowledge in geometry, there are certain cases where the classical framework on differential manifolds does not enable one to work rigorously. It is then necessary to place oneself in a more or less generalized theoretical framework, adapted to the examples studied in order to rigorously prove natural results. One can also (and this happens even in mathematics) work with a formal geometry and invoke “natural properties” in order to overcome difficulties linked to the working formalism. In problems of these kinds, the category of diffeologies offers a coherent framework, easy to use and applicable to most (if not all) cases encountered. This is why we are currently seeing very recent works where diffeologies play a central role, not only in fields related to different geometries (algebraic, topological, differential) but in related fields such as deep learning or optimisation.

It is in this context of strong actual development of research related to diffeologies, both in terms of application and more theoretical aspects, that the special session “Recent advances on diffeologies and their applications” of the AMS-EMS-SMF congress took place in Grenoble, France, from 18 to 20 July 2022. Despite competition from many other leading conferences that were planned at the same time, and a world situation still marked by the pandemic and the sound of cannon fire not far from Western Europe, a rich and varied panel of presentations, presenting different aspects of the subject, stimulated exchanges and future collaborations between participants. Most of them gladly agreed to participate in this volume, to which I have been able to add the contributions of people who were unable to attend for various reasons.

I would therefore like to thank the conference organisers for giving Jordan Watts and me the opportunity to organise this session, and the editors of the Contemporary Mathematics series for this volume, which not only presents some of the most cutting-edge papers on diffeologies, but also an overview of some selected applications of diffeologies to a specific domain. Finally, I must say that I am very grateful to all the contributors for their enthusiastic participation in this volume.

Jean-Pierre Magnot
11 May 2023, Orcines, France

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This volume contains the proceedings of the AMS-EMS-SMF Special Session on Recent Advances in Diffeologies and Their Applications, held from July 18–20, 2022, at the Université de Grenoble-Alpes, Grenoble, France.

The articles present some developments of the theory of diffeologies applied in a broad range of topics, ranging from algebraic topology and higher homotopy theory to integrable systems and optimization in PDE.

The geometric framework proposed by diffeologies is known to be one of the most general approaches to problems arising in several areas of mathematics. It can adapt to many contexts without major technical difficulties and produce examples inaccessible by other means, in particular when studying singularities or geometry in infinite dimension. Thanks to this adaptability, diffeologies appear to have become an interesting and useful language for a growing number of mathematicians working in many different fields. Some articles in the volume also illustrate some recent developments of the theory, which makes it even more deep and useful.



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