

Volume 7

C  
R  
M

CRM  
PROCEEDINGS &  
LECTURE NOTES

Centre de Recherches Mathématiques  
Université de Montréal

Mathematical Quantum  
Theory I: Field Theory  
and Many-Body Theory

J. Feldman  
R. Froese  
L. M. Rosen  
*Editors*



American Mathematical Society

## **Titles in This Series**

### **Volume**

- 7 J. Feldman, R. Froese, and L. M. Rosen, Editors**  
Mathematical quantum theory I: Field theory and many-body theory  
1994
- 6 Guido Mislin, Editor**  
The Hilton Symposium 1993  
Topics in topology and group theory  
1994
- 5 D. A. Dawson, Editor**  
Measure-valued processes, stochastic partial differential equations, and  
interacting systems  
1994
- 4 Hershy Kisilevsky and M. Ram Murty, Editors**  
Elliptic curves and related topics  
1994
- 3 Rémi Vaillancourt and Andrei L. Smirnov, Editors**  
Asymptotic methods in mechanics  
1993
- 2 Philip D. Loewen**  
Optimal control via nonsmooth analysis  
1993
- 1 M. Ram Murty, Editor**  
Theta functions  
1993

*This page intentionally left blank*



# CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques  
Université de Montréal

## Mathematical Quantum Theory I: Field Theory and Many-Body Theory

J. Feldman  
R. Froese  
L. M. Rosen  
*Editors*

The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique.



**American Mathematical Society**  
Providence, Rhode Island USA

The production of this volume was supported in part by the Fonds pour la Formation de Chercheurs et l'Aide à la Recherche (Fonds FCAR) and the Natural Sciences and Engineering Research Council of Canada (NSERC).

1991 *Mathematics Subject Classification*. Primary 81T08; Secondary 81V70.

---

**Library of Congress Cataloging-in-Publication Data**

Mathematical quantum theory I: field theory and many-body theory/ J. Feldman, R. Froese, L. Rosen, editors.

p. cm. — (CRM proceedings & lecture notes, ISSN 1065-8580; v. 7)

Includes bibliographical references (p. ).

ISBN 0-8218-0365-4

I. Quantum field theory—Congresses. 2. Many-body problem—Congresses. I. Feldman, Joel S., 1949- . II. Froese, Richard Gerd. III. Rosen, Lon M., 1944- . IV. Series.

QC174.45.A1M38 1994

530.1'43—dc20

94-37710

CIP

---

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Manager of Editorial Services, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to [reprint-permission@math.ams.org](mailto:reprint-permission@math.ams.org).

The owner consents to copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that a fee of \$1.00 plus \$.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, Massachusetts 01923. When paying this fee please use the code 1065-8580/94 to refer to this publication. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

© Copyright 1994 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights  
except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines  
established to ensure permanence and durability.

♻ Printed on recycled paper.

This volume was typeset using  $\text{AMS-}\text{T}_{\text{E}}\text{X}$ ,  
the American Mathematical Society's  $\text{T}_{\text{E}}\text{X}$  macro system,  
and submitted to the American Mathematical Society in camera-ready  
form by the Centre de Recherches Mathématiques.

10 9 8 7 6 5 4 3 2 1      99 98 97 96 95 94

## Contents

<b>Preface</b>	
<i>Joel S. Feldman, Richard Froese, and Lon M. Rosen</i>	vii
<b>Weak Perturbations of Gaussian Measures</b>	
<i>D. Brydges, J. Dimock, and T. R. Hurd</i>	1
<b>Fermionic Many-Body Models</b>	
<i>Joel Feldman, Jacques Magnen, Vincent Rivasseau, and Eugene Trubowitz</i>	29
<b>Conformal Field Theory and Geometry of Strings</b>	
<i>Jürg Fröhlich and Krzysztof Gawędzki</i>	57
<b>Cluster Expansions with Small/Large Field Conditions</b>	
<i>Vincent Rivasseau</i>	99
<b>Percolation Theory and the Phase Structure of Two-Dimensional Classical Ferromagnets</b>	
<i>Adrian Patrascioiu and Erhard Seiler</i>	153
<b>Navier and Stokes Meet the Wavelet, II</b>	
<i>Paul Federbush</i>	163
<b>Applications of the Renormalization Group</b>	
<i>D. Brydges, J. Dimock, and T. R. Hurd</i>	171
<b>End-to-End Distance for a Four-Dimensional Self-Avoiding Walk</b>	
<i>John Z. Imbrie</i>	191
<b>The Falicov-Kimball Model of Interacting Electrons</b>	
<i>Tom Kennedy</i>	197
<b>Radiative Decay of an Atom in a Massless Quantised Field</b>	
<i>Christopher King</i>	203
<b>The Knizhnik-Zamolodchikov-Bernard Equation on the Torus</b>	
<i>G. Felder and C. Wierczkowski</i>	211
<b>Dual Potentials of Free Dirac Currents as Exactly Soluble Models</b>	
<i>Arthur S. Wightman</i>	221

*This page intentionally left blank*

## Preface

These articles constitute the proceedings of the Canadian Mathematical Society Annual Seminar on Mathematical Quantum Theory held in Vancouver August 4–14, 1993. The meeting was run as a research level summer school concentrating on two related areas of contemporary mathematical physics. The subject for the first session was quantum field theory and many-body theory and is covered in Volume 1 of these proceedings. The second session dealt with Schrödinger operators and is covered in Volume 2. Each session featured a series of mini-courses, consisting of approximately four one hour lectures, designed to introduce the student to the state of the art in a particular area. In addition, about thirty speakers gave one hour lectures of an expository nature.

The meeting could not have taken place without the sponsorship of the Natural Sciences and Engineering Research Council (NSERC) and the Centre de Recherches Mathématiques (CRM) of the Université de Montréal. We also wish to acknowledge the assistance of the Canadian Mathematical Society and the Department of Mathematics of the University of British Columbia, in particular Nina Einarsson and Margaret Harney.

Joel Feldman  
Richard Froese  
Lon Rosen

August 1993



*This page intentionally left blank*

ISBN 0-8218-0365-4



9 780821 803653