

Volume 12

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Centre de Recherches Mathématiques  
Université de Montréal

Partial Differential  
Equations and Their  
Applications

Peter C. Greiner  
Victor Ivrii  
Luis A. Seco  
Catherine Sulem  
*Editors*



American Mathematical Society

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Partial Differential  
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Volume 12



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# CRM PROCEEDINGS & LECTURE NOTES

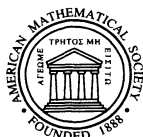
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Centre de Recherches Mathématiques  
Université de Montréal

## Partial Differential Equations and Their Applications

Peter C. Greiner  
Victor Ivrii  
Luis A. Seco  
Catherine Sulem  
*Editors*

The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique.



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## Preface

This volume collects the lectures given at the 1995 Annual Seminar of the Canadian Mathematical Society devoted to Partial Differential Equations and its Applications, and hosted by the University of Toronto during the unusually warm days of June 12-23, 1995.

The conference consisted of a combination of minicourses, invited presentations and contributed talks. This volume is an attempt to create a historical record of that event, by including contributions on a variety of topics related to PDE such as spectral asymptotics, harmonic analysis, differential operators in hyperbolic manifolds, applications to geometry, mathematical physics, hydrodynamics and interaction between theory and numerical methods in PDE.

The conference attracted a large number of mathematicians and students, as well as scientists in Physics and related disciplines, all of whom contributed to make this mathematical gathering into a memorable scientific experience.

We are grateful to the Canadian Mathematical Society, the National Science and Engineering Research Council of Canada, the Fields Institute for Research in Mathematical Sciences, the Centre de recherches mathématiques and the Mathematics Department at the University of Toronto for their financial and administrative support, and for the publication of these proceedings.

We wish to express our special thanks to the lecturers for their stimulating presentations and to all the participants who contributed to the success of the meeting.

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