

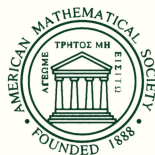


# FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

## Novel Approaches to Hard Discrete Optimization

Panos Pardalos  
Henry Wolkowicz  
Editors



**American Mathematical Society**

Novel Approaches  
to Hard Discrete  
Optimization

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**American Mathematical Society**  
Providence, Rhode Island

## The Fields Institute for Research in Mathematical Sciences

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## Preface

This volume contains refereed papers presented at the workshop on “Novel Approaches to Hard Discrete Optimization” held at the University of Waterloo during April 26 – 28, 2001. The workshop was supported by the Faculty of Mathematics at the University of Waterloo, CRM, and the Fields Institute, Canada.

During the last decade, many novel approaches have been considered to deal with computationally difficult discrete optimization problems. Such approaches, include interior point methods, semidefinite programming techniques, and global optimization. More efficient computational algorithms have been developed and larger problem instances of hard discrete problems have been solved. This progress is due in part to these novel approaches, but also to new computing facilities such as massive parallelism.

The papers in this volume cover a spectrum of issues regarding computationally hard discrete problems. The paper by Barvinok and Stephen deals with the distribution values of the quadratic assignment problem. This is a difficult problem and only recently a major break-through was achieved to solve QAP of sizes  $n = 30$ . Boginski et al discuss optimization problems in massive graphs. Examples of optimization problems in massive graphs arising in telecommunications, Internet, and finance are presented. The paper by Cardei et al presents results on the famous guillotine cut problem. Cheng et al present three approximation algorithms for wavelength assignment in multifiber networks. The paper by Coppersmith and Lee deals with indivisibility and divisibility polytopes. Hager’s paper presents the dual set algorithm applied to linear programming. Hillar and Johnson study some eigenvalue problems. Semi-infinite linear programming techniques for solving semidefinite problems is the focus of the paper by Krishnan and Mitchell. The paper by Lasserre deals with polynomial optimization. Semidefinite programming versus linear programming relaxations are considered. Min et al present approximation algorithms for the Steiner minimum tree problem in the presence of obstructions. The paper by Mokhtarian deals with a convex feasibility problem defined by a nonlinear separation obstacle. Finally, the paper by Zhou et al presents algorithms for the smallest enclosing ball problem in higher dimensions. The volume would be suitable as a complimentary material for advanced courses in combinatorial optimization.

We would like to take this opportunity to thank the authors of the papers, the referees, The Fields Institute, and the Faculty of Mathematics of the University of Waterloo for the support and help with the workshop.

Panos M. Pardalos and Henry Wolkowicz  
University of Florida and University of Waterloo  
November 2002



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This volume contains the papers presented at the workshop on “Novel Approaches to Hard Discrete Optimization”. The articles cover a spectrum of issues regarding computationally hard discrete problems. The volume is suitable for graduate students and research mathematicians interested in theoretical and computational aspects of optimization.

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