FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Galois Theory, Hopf Algebras, and Semiabelian Categories

George Janelidze
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Editors

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The Fields Institute for Research in Mathematical Sciences

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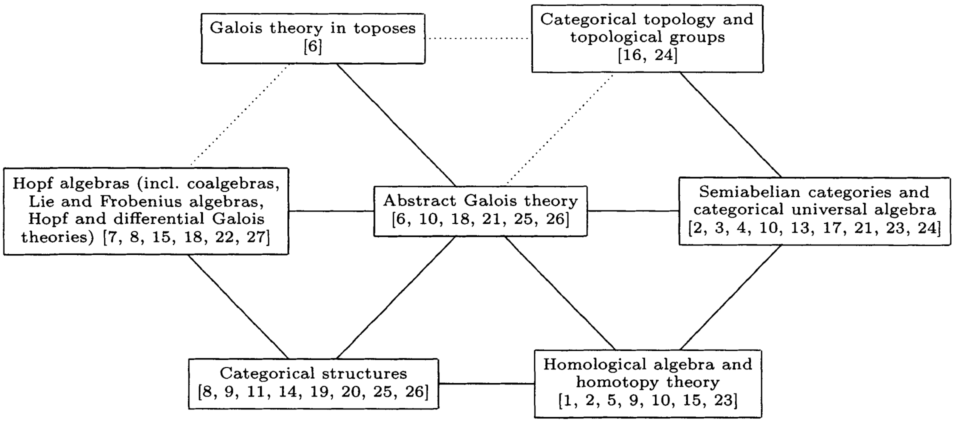
Preface

During the week of September 23–28, 2002, the editors of this volume organized a *Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras, and Semiabelian Categories* at the Fields Institute for Research in Mathematical Sciences in Toronto. The goal of the Workshop was to bring together researchers working in the quite distinct but nevertheless interrelated and partly overlapping areas mentioned in its title. The meeting was attended by almost eighty mathematicians from various research communities and boasted twenty invited lectures and numerous contributed talks that led to an inspiring atmosphere of learning and scientific exchange.

This volume can only partially reflect the Workshop's themes but should nevertheless give the reader a good idea about the current connections among abstract Galois theories, Hopf algebras, and semiabelian categories. Here is a very brief indication of the origins of those connections. Hopf algebras arrived to the Galois theory of rings as early as the 1960s — independently of, but in fact similarly to, the way in which algebraic group schemes were introduced to the theory of étale coverings in algebraic geometry. Galois theory, in turn, was extended to elementary toposes and was then formulated in purely categorical contexts. Eventually it became general enough to even include abstractions of the theory of central extensions, to mention only one of various fairly recent developments. In fact, classically, central extensions of groups together with the homology functors $H_1(-,\mathbf{Z})$ and $H_2(-,\mathbf{Z})$ can be used to begin homological algebra, just like covering spaces together with the homotopy functors π_0 and π_1 are the starting gadgets of homotopy theory. Finally, during the past four years semiabelian categories have emerged as a very good environment in which to pursue not just basic modern algebra but in fact homological algebra of groups and other non-abelian structures categorically.

Given the diversity of the backgrounds of the presenters at the Workshop, this volume cannot be expected to contain a homogeneous sequence of chapters on its themes. Rather, the reader will find a collection of beautiful but fairly independent articles on selected topics in algebra, topology, and pure category theory that should seriously contribute to the categorical unification of the subjects in question. The survey articles contained in this volume should be particularly helpful in this regard.

A rough general "map" of the topics/articles presented in this volume may be displayed as follows, with the numbers referring to the (alphabetical) list of contributions contained in the volume. Most of the papers are mentioned more than once. Solid lines represent links explicitly discussed in this volume, while dotted lines indicate other known links.



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George Janelidze
Bodo Pareigis
Walter Tholen

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This volume is based on talks given at the Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras, and Semiabelian Categories held at The Fields Institute for Research in Mathematical Sciences (Toronto, ON, Canada). The meeting brought together researchers working in these interrelated areas.

This collection of survey and research papers gives an up-to-date account of the many current connections among Galois theories, Hopf algebras, and semiabelian categories. The book features articles by leading researchers on a wide range of themes, specifically, abstract Galois theory, Hopf algebras, and categorical structures, in particular quantum categories and higher-dimensional structures.

Articles are suitable for graduate students and researchers, specifically those interested in Galois theory and Hopf algebras and their categorical unification.

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