



FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Geometry and Topology of Manifolds

Hans U. Boden
Ian Hambleton
Andrew J. Nicas
B. Doug Park
Editors



American Mathematical Society

Geometry and Topology of Manifolds

This page intentionally left blank



FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Geometry and Topology of Manifolds

Hans U. Boden
Ian Hambleton
Andrew J. Nicas
B. Doug Park
Editors



American Mathematical Society
Providence, Rhode Island

The Fields Institute for Research in Mathematical Sciences

The Fields Institute is a center for mathematical research activity, located in Toronto, Canada. Our mission is to provide a supportive and stimulating environment for mathematics research, innovation and education. The Institute is supported by the Ontario Ministry of Training, Colleges and Universities, the Natural Sciences and Engineering Research Council of Canada, and seven Ontario universities (Carleton, McMaster, Ottawa, Toronto, Waterloo, Western Ontario, and York). In addition there are several affiliated universities and corporate sponsors in both Canada and the United States.

Fields Institute Editorial Board: Carl R. Riehm (Managing Editor), Barbara Lee Keyfitz (Director of the Institute), Thomas S. Salisbury (Deputy Director), John Bland (Toronto), Kenneth R. Davidson (Waterloo), Joel Feldman (UBC), R. Mark Goresky (Institute for Advanced Study, Princeton), Cameron Stewart (Waterloo), Noriko Yui (Queen's).

2000 *Mathematics Subject Classification*. Primary 57-06, 53-06, 58-06, 14-06;
Secondary 57M27, 57R58, 57R57, 57R17, 57M25, 57N13, 53D45 14N35, 32S25, 57M50,
57R15, 57R55, 57R40, 57R65, 57R67, 57R80, 57M60, 57N10, 53D35, 53D40, 58D15,
58D27, 58D29, 14J28, 14E15, 14B15, 14J17, 32S45.

Library of Congress Cataloging-in-Publication Data

Geometry and topology of manifolds / Hans U. Boden...[et al.], editors.

p. cm. — (Fields Institute communications, ISSN 1069-5265 ; 47)

Includes bibliographical references.

ISBN 0-8218-3724-9 (acid-free paper)

I. Topology—Congresses. 2. Manifolds (Mathematics)—Congresses. I. Boden, Hans U., 1962–
II. Fields Institute communications ; v. 47.

QA611.A1.G47 2005
514—dc22

2005048294
CIP

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2005 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Copyright of individual articles may revert to the public domain 28 years
after publication. Contact the AMS for copyright status of individual articles.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

This publication was prepared by The Fields Institute.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 10 09 08 07 06 05

Contents

Preface	vii
Conference Speakers	ix
Conference Participants	xi
An Involution Acting Nontrivially on Heegaard-Floer Homology SELMAN AKBULUT and SELAHI DURUSOY	1
Pseudoholomorphic Curves in Four-Orbifolds and Some Applications WEIMIN CHEN	11
Floer Homology for Knots and 3-Manifolds and Cyclic Dehn Surgeries Along Knots OLIVIER COLLIN	39
A $\mathrm{PSL}_2(\mathbb{C})$ Casson Invariant CYNTHIA L. CURTIS	51
The Borel/Novikov Conjectures and Stable Diffeomorphisms of 4-Manifolds JAMES F. DAVIS	63
Invariants of Knots, Embeddings, and Immersions via Contact Geometry TOBIAS EKHOLM and JOHN B. ETNYRE	77
$\mathrm{SO}(3)$ Monopoles: The Overlap Problem PAUL M. N. FEEHAN AND THOMAS G. LENESE	97
Heegaard Floer Homology of Mapping Tori II STANISLAV JABUKA and THOMAS MARK	119
Surfaces in 4-Manifolds and the Surgery Conjecture VYACHESLAV S. KRUSHKAL	137
Symplectic Gluing and Family Gromov-Witten Invariants JUNHO LEE and THOMAS H. PARKER	147

Heegaard Splittings and Seiberg-Witten Monopoles YI-JEN LEE	173
Existence of Symplectic Surfaces TIAN-JUN LI	203
On the Heegaard Floer homology of $S^3_{-d}(K)$ and Unicuspidal Rational Plane Curves ANDRÁS NÉMETHI	219
Examples of Symplectic 4-Manifolds with Positive Signature MARTIN NIEPEL	235
Definite Manifolds Bounded by Rational Homology Three Spheres BRENDAN OWENS and SAŠO STRLE	243
On Park's Exotic Smooth Four-Manifolds PETER OZSVÁTH and SOLTÁN SZABÓ	253
Knot Polynomials and Knot Homologies JACOB RASMUSSEN	261
Casson-Type Invariants in Dimension Four DANIEL RUBERMAN and NIKOLAI SAVELIEV	281
Culler-Shalen Norms and Invariant Trace Fields SAMUEL SCHACK and XINGRU ZHANG	307
Complex Spin Structures on 3-Manifolds LAURENCE R. TAYLOR	313
Lefschetz Fibrations and Pseudoholomorphic Curves MICHAEL USHER	319
The Isotopy Problem for Symplectic 4-Manifolds STEFANO VIDUSSI	335

Preface

The papers collected for this volume were contributed by participants of the Conference “Geometry and Topology of Manifolds” held at McMaster University, May 14–18, 2004, and they are indicative of the many excellent talks delivered there. A common theme at the meeting was the problem of understanding smooth manifolds in dimensions three and four, and the papers in this volume represent the many diverse approaches one can take in studying this problem. Several of the papers on knot theory and 3-manifolds feature the Heegaard-Floer homology of Ozsváth and Szabó, while others develop techniques and explore results using a variety of methods such as algebraic topology, surgery theory, gauge theory (both Donaldson and Seiberg-Witten), contact and symplectic geometry, and Gromov-Witten invariants.

This conference was the natural successor to the 1990 conference “Topology of Four-Manifolds,” also held at McMaster. The Four-Manifolds Conference is memorable as the setting where Bob Gompf and Tom Mrowka discovered the first examples of exotic smooth structures on complex surfaces, and their discovery refuted a well-known conjecture that simply connected 4-manifolds could all be written as connected sums of complex surfaces. Exotic smooth structures continue to be a topic of great current interest, indeed one of the highlights of the meeting was the talk delivered by Jongil Park outlining his construction of an exotic smooth structure on $\mathbb{C}P^2 \# 7\overline{\mathbb{C}P^2}$ (cf. the paper by Ozsváth and Szabó below). In the past year, there has been a flurry of activity on this problem, and currently the best result, due to Park, Stipsicz, and Szabó, gives the existence of infinitely many differentiable structures on $\mathbb{C}P^2 \# n\overline{\mathbb{C}P^2}$ for $n \geq 5$ (see the recent survey article [math.GT 0502164](#) by Ron Stern for more details). Another highlight of the conference was the talk by Tom Mrowka, where he presented his recent joint work with Peter Kronheimer proving Property P for all knots. It is truly remarkable how far gauge theory and low-dimensional topology have advanced since the Four-Manifolds Conference 15 years ago!

We would like to take this opportunity to thank all the people who helped in organizing the conference and compiling this book. The conference was funded by the Fields Institute and by McMaster University, and we would like to express our sincere appreciation to Ken Davidson, Director of the Fields Institute, and to Peter Sutherland, Dean of the Faculty of Science at McMaster University, for their help in making the conference a success. We are also grateful to the staff at Fields and the staff in the Department of Mathematics and Statistics at McMaster for their friendly assistance, especially Alison Conway and Andrea Hartin for their administrative support, and Tina Campbell, Diane Holmes, and Barbara Ulitsky for their logistical support. Finally, we would like to thank Debbie Iscoe for her TeX-expertise, without which this volume would never have seen the light of day!

The Editors

This page intentionally left blank

Conference Speakers

SELMAN AKBULUT	<i>4-manifold corks and fillings</i>
DENIS AUROUX	<i>Near symplectic structures and Lefschetz pencils on smooth 4-manifolds</i>
SCOTT BALDRIDGE	<i>Geography of symplectic 4-manifolds with Kodaira dimension one</i>
IGOR BELEGRADEK	<i>Collapsing theory and classification of negatively pinched manifolds with amenable fundamental groups</i>
STEVE BOYER	<i>Characteristic subsurfaces, character varieties and Dehn filling</i>
WEIMIN CHEN	<i>Smooth s-cobordisms of elliptic 3-manifolds</i>
JIM DAVIS	<i>Stable Diffeomorphisms and a fake connected sum of $\mathbb{R}P^4$</i>
JOHN ETNYRE	<i>Invariants of embeddings via contact geometry</i>
PAUL FEEHAN	<i>$SO(3)$ monopoles: The overlap problem I</i>
RON FINTUSHEL	<i>Invariants of Lagrangian tori</i>
KIM FRØYSHOV	<i>Compactness and non-compactness for monopoles</i>
KO HONDA	<i>Right-veering diffeomorphisms of a compact surface with boundary</i>
ELENY IONEL	<i>Curves and Gromov-Witten invariants of 3-folds</i>
STANISLAV JABUKA	<i>Heegaard Floer homology and mapping tori I</i>
PAUL KIRK	<i>Minimizing the Euler characteristic of 4-manifolds with free abelian fundamental groups</i>
SLAVA KRUSHKAL	<i>Link groups of 4-manifolds</i>
YI-JEN LEE	<i>A Seiberg-Witten model for Heegaard Floer homologies</i>
TOM LENESE	<i>$SO(3)$ monopoles: The overlap problem II</i>
TIAN-JUN LI	<i>Symplectic surfaces in symplectic manifolds</i>
THOMAS MARK	<i>Heegaard Floer homology and mapping tori II</i>
TOMASZ MROWKA	<i>From foliations of three manifolds to representations of their fundamental groups</i>
ANDRÁS NÉMETHI	<i>The Seiberg-Witten invariant conjecture and projective plane curves</i>
BRENDAN OWENS	<i>Definite manifolds bounded by rational homology three-spheres I</i>
JONGIL PARK	<i>New symplectic 4-manifolds with $b_2^+ = 1$</i>
JACOB RASMUSSEN	<i>Khovanov homology and the slice genus</i>
DANNY RUBERMAN	<i>Rohlin's invariant and gauge theory I</i>
NIKOLAI SAVELIEV	<i>Rohlin's invariant and gauge theory II</i>
SAŠO STRLE	<i>Definite manifolds bounded by rational homology three-spheres II</i>
MICHAEL USHER	<i>Lefschetz fibrations and pseudoholomorphic curves</i>
STEFANO VIDUSSI	<i>Isotopy problem for symplectic 4-manifolds</i>

This page intentionally left blank

Conference Participants

Danny Acosta, Southeastern Louisiana University
Selman Akbulut, Michigan State University
Dave Auckly, Kansas State University
Denis Auroux, Massachusetts Institute of Technology
Scott Baldrige, Indiana University
Igor Belegradek, Georgia Institute of Technology
Hans U. Boden, McMaster University
Steve Boyer, Université du Québec à Montréal
Vasile Brinzanescu, Romanian Academy
Olguta Buse, Michigan State University
Jason Calmes, Tulane University
Weimin Chen, Tulane University
Patrick Clarke, University of Miami
Olivier Collin, Université du Québec à Montréal
Jim Davis, Indiana University
Fabrizio Donzelli, University of Miami
George Dragomir, McMaster University
Tolga Etgü, McMaster University
John Etnyre, University of Pennsylvania
Paul Feehan, Rutgers University
Praphat Fernandes, McMaster University
Ronald Fintushel, Michigan State University
Stefan Friedl, Rice University
Kim Frøyshov, University of Bielefeld
David Gay, Université du Québec à Montréal
Robert Gompf, University of Texas, Austin
Pengfei Guan, McMaster University
Ian Hambleton, McMaster University
Mathew Hedden, Columbia University
Christopher Herald, University of Nevada, Reno
Benjamin Himpel, Indiana University
Ko Honda, University of Southern California
Jacques Hurtubise, McGill University
Gabriel Indurskis, Université du Québec à Montréal
Eleny Ionel, University of Wisconsin
Marianty Ionel, McMaster University
Stanislav Jabuka, Columbia University
Lisa Jeffrey, University of Toronto
Spiro Karigiannis, McMaster University
Paul Kirk, Indiana University

Vyacheslav Krushkal, University of Virginia
Terry Lawson, Tulane University
Yi-Jen Lee, Institute for Advanced Study
Tom Leness, Florida International University
Tian-Jun Li, University of Minnesota
Yuhan Lim, University of California, Santa Cruz
Natasha Macura, Trinity University
Thomas Mark, Southeastern Louisiana University
Joseph Masters, University of Buffalo
Gordana Matic, University of Georgia
Mike McCooey, Franklin and Marshall College
William Menasco, University of Buffalo
Maung Min-Oo, McMaster University
Ruxandra Moraru, Fields Institute
Tomasz Mrowka, Massachusetts Institute of Technology
András Némethi, Ohio State University
Andy Nicas, McMaster University
Liviu Nicolaescu, University of Notre Dame
Martin Niepel, Princeton University
Brendan Owens, McMaster University
Burak Ozbagci, Koç University
Mehmetcik Pamuk, McMaster University
Semra Pamuk, McMaster University
B. Doug Park, University of Waterloo
Jongil Park, Konkuk University
Tom Parker, Michigan State University
Eric Pedersen, SUNY Binghamton
Mainak Poddar, University of Waterloo
Andrew Ranicki, University of Edinburgh
Jacob Rasmussen, Princeton University
Dale Rolfsen, University of British Columbia
David Rosenthal, McMaster University
Daniel Ruberman, Brandeis University
Nikolai Saveliev, University of Miami
Adam Sikora, University of Buffalo
Sašo Strle, McMaster University
Margaret Symington, Georgia Institute of Technology
Masamichi Takase, Yokohama National University
Laurence Taylor, University of Notre Dame
Barbara Ulitsky, McMaster University
Michael Usher, Massachusetts Institute of Technology
Jeremy van Horn, University of Texas, Austin
Stefano Vidussi, Kansas State University
Jens von Bergmann, Michigan State University
McKenzie Wang, McMaster University
Wojciech Wieczorek, University of Wisconsin
Xingru Zhang, University of Buffalo

Titles in This Series

- 47 **Hans U. Boden, Ian Hambleton, Andrew J. Nicas, and B. Doug Park, Editors,** Geometry and topology of manifolds, 2005
- 46 **Thierry Passot, Catherine Sulem, and Pierre-Louis Sulem, Editors,** Topics in kinetic theory, 2005
- 45 **Ragnar-Olaf Buchweitz and Helmut Lenzing, Editors,** Representations of algebras and related topics, 2005
- 44 **Lajos Horváth and Barbara Szyszkowicz, Editors,** Asymptotic methods in stochastics, 2004
- 43 **George Janelidze, Bodo Pareigis, and Walter Tholen, Editors,** Galois theory, Hopf algebras, and semiabelian categories, 2004
- 42 **Saber Elaydi, Gerry Ladas, Jianhong Wu, and Xingfu Zou, Editors,** Difference and differential equations, 2004
- 41 **Alf van der Poorten and Andreas Stein, Editors,** High primes and misdemeanours: Lectures in honour of the 60th birthday of Hugh Cowie Williams, 2004
- 40 **Vlastimil Dlab and Claus Michael Ringel, Editors,** Representations of finite dimensional algebras and related topics in Lie theory and geometry, 2004
- 39 **Stephen Berman, Yuly Billig, Yi-Zhi Huang, and James Lepowsky, Editors,** Vector operator algebras in mathematics and physics, 2003
- 38 **Noriko Yui and James D. Lewis, Editors,** Calabi-Yau varieties and mirror symmetry, 2003
- 37 **Panos Pardalos and Henry Wolkowicz, Editors,** Novel approaches to hard discrete optimization, 2003
- 36 **Shigui Ruan, Gail S. K. Wolkowicz, and Jianhong Wu, Editors,** Dynamical systems and their applications in biology, 2003
- 35 **Yakov Eliashberg, Boris Khesin, and François Lalonde, Editors,** Symplectic and contact topology: Interactions and perspectives, 2003
- 34 **T. J. Lyons and T. S. Salisbury, Editors,** Numerical methods and stochastics, 2002
- 33 **Franz-Viktor Kuhlmann, Salma Kuhlmann, and Murray Marshall, Editors,** Valuation theory and its applications, Volume II, 2003
- 32 **Franz-Viktor Kuhlmann, Salma Kuhlmann, and Murray Marshall, Editors,** Valuation theory and its applications, Volume I, 2002
- 31 **A. Galves, J. K. Hale, and C. Rocha, Editors,** Differential equations and dynamical systems, 2002
- 30 **Roberto Longo, Editor,** Mathematical physics in mathematics and physics: Quantum and operator algebraic aspects, 2001
- 29 **Teresa Faria and Pedro Freitas, Editors,** Topics in functional differential and difference equations, 2001
- 28 **David R. McDonald and Stephen R. E. Turner, Editors,** Analysis of communication networks: Call centres, traffic and performance, 2000
- 27 **Shui Feng and Anna T. Lawniczak, Editors,** Hydrodynamic limits and related topics, 2000
- 26 **Neal Madras, Editor,** Monte Carlo methods, 2000
- 25 **A. G. Ramm, P. N. Shivakumar, and A. V. Strauss, Editors,** Operator theory and its applications, 2000
- 24 **Edward Bierstone, Boris Khesin, Askold Khovanskii, and Jerrold E. Marsden, Editors,** The Arnoldfest: Proceedings of a conference in honour of V. I. Arnold for his sixtieth birthday, 1999

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/.

This book contains expository papers that give an up-to-date account of recent developments and open problems in the geometry and topology of manifolds, along with several research articles that present new results appearing in published form for the first time. The unifying theme is the problem of understanding manifolds in low dimensions, notably in dimensions three and four, and the techniques include algebraic topology, surgery theory, Donaldson and Seiberg-Witten gauge theory, Heegaard Floer homology, contact and symplectic geometry, and Gromov-Witten invariants. The articles collected for this volume were contributed by participants of the Conference “Geometry and Topology of Manifolds” held at McMaster University on May 14-18, 2004 and are representative of the many excellent talks delivered at the conference.

ISBN 0-8218-3724-9



9 780821 837245

FIC/47

AMS *on the Web*
www.ams.org