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Holomorphic Dynamics and Renormalization

A Volume in Honour of John Milnor's 75th Birthday

Mikhail Lyubich
Michael Yampolsky

Editors
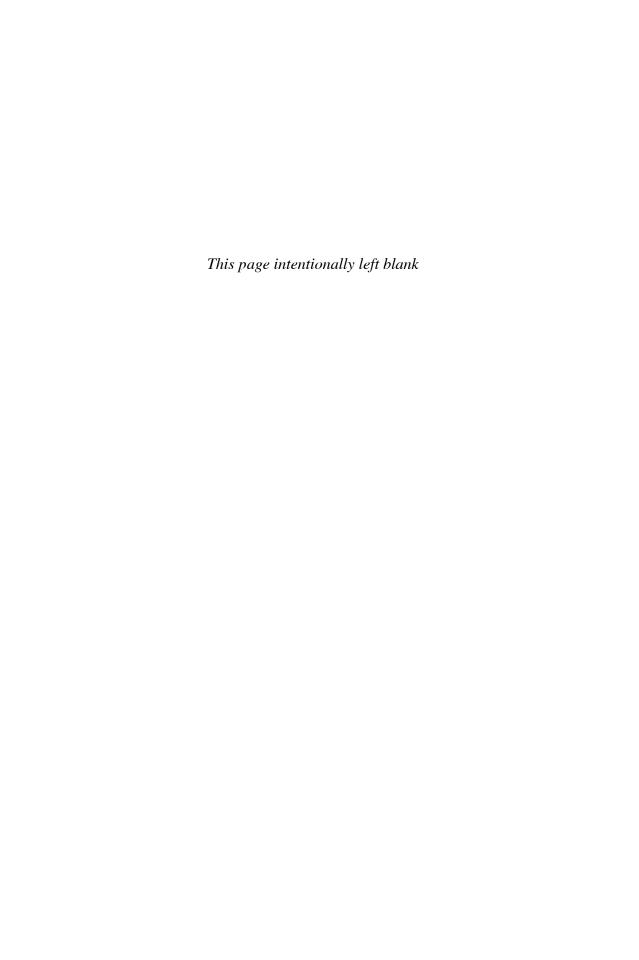
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THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

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and Renormalization

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The Fields Institute
for Research in Mathematical Sciences

Contents

Renormalization

Preface

Through the academic year 2005-2006, the Fields Institute ran two programs in dynamics and related areas: "Renormalization and universality in mathematics and mathematical physics" in the fall and "Holomorphic dynamics, laminations and hyperbolic geometry"in the spring. A festive highlight of the year was a Workshop on Holomorphic Dynamics, which was held on March 7–11, 2006 to celebrate John Milnor's 75th birthday. Three generations of dynamicists gathered at the Fields Institute to take stock of the explosive development the subject has experienced since the early 1980's. Closely related themes of Renormalization in Dynamics were discussed at the Workshop held in November 2005.

The papers collected in this volume reflect some of the directions of research presented at the two Workshops. Below we will provide the reader with a brief (and incomplete) guide to the main themes.

The volume opens with a paper by A. Bonifant and J. Milnor on dynamics of fibered cylinder maps. It shows that the observed dynamical features depend strongly on the sign of the fibered Schwarzian derivative.

Dynamics of polynomial mappings, particularly quadratics, has by now reached a mature state of development. By comparison, much less is known about non-polynomial rational maps. In particular, some of the combinatorial tools, such as puzzle partitions, which have been so powerful for polynomial maps, have not yet been adequately developed in the non-polynomial setting. Description of parameter spaces, even in special cases, is a challenging problem. The reader will be led into the world of non-polynomial rational dynamics by the papers of R. Devaney and V. Timorin in this volume.

Transcendental dynamics exhibits many common features, as well as significant differences, with polynomial dynamics. A survey of L. Rempe and D. Schleicher presents a comparison of the two fields. Finding roots of entire functions using Newton's method leads to iteration of a meromorphic map of the Riemann sphere. Here methods of complex dynamics beautifully intertwine with root-finding algorithms, as seen in the papers by J. Rückert and D. Schleicher in this volume. The latter paper contains amusing applications to search of the roots of the Riemann ζ-function.

The volume contains a survey by V. Nekrashevych on Iterated Monodromy Groups of rational mappings. This subject links geometric group theory to combinatorics of rational maps. Translated into this language, many questions related to Thurston's theory of branched coverings of the sphere, such as the "twisted rabbit problem" can be answered explicitly.

Another approach to understanding of covering properties of polynomials based on the "Green tree" of the basin at infinity is presented in the paper by L. DeMarco.

First introduced into the field by Douady and Hubbard and Sullivan in mid 1980's, Renormalization Theory now occupies a central place in modern Complex
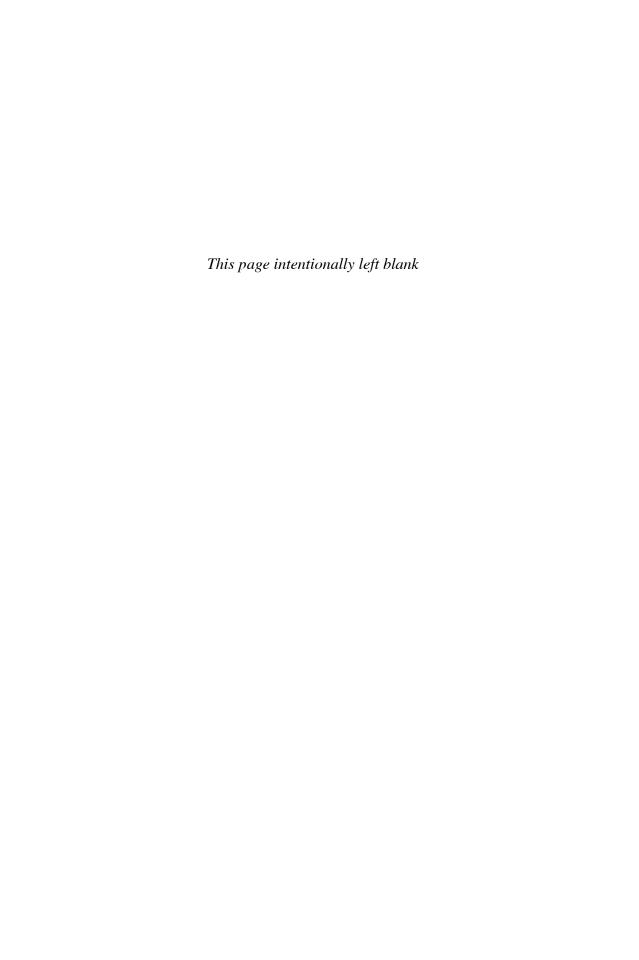
Dynamics. The progress in understanding the structure of the Mandelbrot set, polynomial Julia sets, and Feigenbaum-type universalities stems from renormalization techniques. A central open problem of MLC (local connectedness of the Mandelbrot set) has been connected to renormalization theory through the work of Yoccoz.

Renormalization of critical circle maps and Siegel disks has attracted much attention recently, due to a breakthrough by Inou-Shishikura, and subsequent work of Buff-Chéritat, which completed Douady's Program of constructing Julia sets with positive area. The paper of M. Yampolsky in this volume develops a renormalization theory of Siegel disks, its relation to the work of Inou-Shishikura, and the connection with Douady's Program.

Renormalization of circle maps and rotation domains can be understood in the context of the classical KAM theory. Corresponding phenomena in higher dimensions, such as universal scaling in area-preserving maps in 2D, on the boundary of KAM, still pose a challenging problem. An approach to it, based on renormalization of Hamiltonian vector fields, has come to fruition in the work of H. Koch. This volume contains his survey on the subject, which introduces the reader to these important questions, and describes the current state-of-the-art.

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Mikhail Lyubich
Michael Yampolsky
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Titles in This Series

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The papers collected in this volume reflect some of the directions of research in two closely related fields: Complex Dynamics and Renormalization in Dynamical Systems.

While dynamics of polynomial mappings, particularly quadratics, has by now reached a mature state of development, much less is known about non-polynomial rational maps. The reader will be introduced into this fascinating world and a related area of transcendental dynamics by the papers in this volume. A graduate student will find an area rich with open problems and beautiful computer simulations.

A survey by V. Nekrashevych introduces the reader to iterated monodromy groups of rational mappings, a recently developed subject that links geometric group theory to combinatorics of rational maps. In this new language, many questions related to Thurston's theory of branched coverings of the sphere can be answered explicitly.

Renormalization theory occupies a central place in modern Complex Dynamics. The progress in understanding the structure of the Mandelbrot set, polynomial Julia sets, and Feigenbaum-type universalities stems from renormalization techniques. Renormalization of circle maps and rotation domains, such as Siegel disks, can be understood in the context of the classical KAM theory. Corresponding phenomena in higher dimensions, such as universal scaling in area-preserving maps in 2D, on the boundary of KAM, pose a challenging problem. A survey by H. Koch and several other papers in the volume will introduce the reader to this direction of study.