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Terry A. Loring



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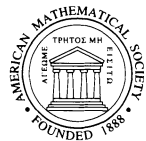


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Lifting Solutions to Perturbing Problems in C^* -Algebras

Terry A. Loring



American Mathematical Society
Providence, Rhode Island

The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

The Fields Institute for Research in Mathematical Sciences is supported by grants from the Ontario Ministry of Education and Training and the Natural Sciences and Engineering Research Council of Canada. The Institute is sponsored by McMaster University, the University of Toronto, the University of Waterloo, and York University and has affiliated universities in Ontario and across Canada.

Some of the author's research summarized in this volume was supported by the National Science Foundation (DMS #9215024).

ABSTRACT. The theory of C^* -algebras determined by generators and relations is developed, with emphasis on the stability properties of those relations. Many lifting and perturbing problems are solved, including problems related to the classification of AD algebras and to almost commuting matrices.

1991 *Mathematics Subject Classification*. Primary 46L05;
Secondary 16D25, 47B15, 41A36.

Library of Congress Cataloging-in-Publication Data

Loring, Terry A., 1959–

Lifting solutions to perturbing problems in C^* -algebras / Terry A. Loring.

p. cm. — (Fields Institute monographs, ISSN 1069-5273 ; 8)

Includes bibliographical references (p. —) and index.

ISBN 0-8218-0602-5 (alk. paper)

1. C^* -algebras. I. Title. II. Series.

QA326.L67 1997

512'.55—dc20

96-43189

CIP

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This publication was prepared by the Fields Institute.

10 9 8 7 6 5 4 3 2 02 01 00 99 98

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Preface

This book began as an attempt to pull together the many papers that comprise the subject called “stable relations for C^* -algebras.” This is a type of perturbation theory for elements of a C^* -algebra. The nature of C^* -algebras is such that one cannot study perturbation without also studying the theory of lifting and the theory of extensions.

The approach taken to these subjects is highly algebraic. This is especially true of the preliminary material in part I. Many (apparently) new results are included. Some of these have no good analog in either topology or in the theory of unital rings. I have developed many results in the context of “ σ -unital” rings to make clear that these are essentially algebraic results which happen to only be true, or only be interesting, for rings that behave very much like C^* -algebras.

In the event the reader has an adverse reaction to algebra, an introduction is included to explain where all the algebra is to be used.

The reader is assumed to have digested the material on C^* -algebras covered in Murphy [1990]. In a few places this is not sufficient, but I have stated the needed result and leave it to the reader to take its veracity on faith or search the designated reference.

I would like to thank those who endured the very early drafts and those who answered many questions or made suggestions: Cristina Cerri, Søren Eilers, Marius Dadarlat, Ken Davidson, Peter Friis, Ken Goodearl, Don Hadwin, Martha Monteiro, Gert Pederson and Jack Spielberg.

Some of the research expositied herein was conducted at the fine facilities of The Fields Institute.

During the past year, the topic of stable relations has become very active. So that this book might appear while still timely, it has been necessary to stay mute on some of the latest results. I will collect recent references to papers on stable relations, after-the-last-minute observations and corrections on my homepage: <http://www.math.unm.edu/~loring>

Terry A. Loring, April 1996

With just enough learning to misquote.

—Lord Byron

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Lifting Solutions to Perturbing Problems in C^* -Algebras

Terry A. Loring

The nature of C^* -algebras is such that one cannot study perturbation without also studying the theory of lifting and the theory of extensions. Approximation questions involving representations of relations in matrices and C^* -algebras are the central focus of this volume. A variety of approximation techniques are unified by translating them into lifting problems: from classical questions about transitivity of algebras of operators on Hilbert spaces to recent results in linear algebra. One chapter is devoted to Lin's theorem on approximating almost normal matrices by normal matrices.

The techniques of universal algebra are applied to the category of C^* -algebras. An important difference, central to this book, is that one can consider approximate representations of relations and approximately commuting diagrams. Moreover, the highly algebraic approach does not exclude applications to very geometric C^* -algebras.

K -theory is avoided, but universal properties and stability properties of specific C^* -algebras that have applications to K -theory are considered. Index theory arises naturally, and very concretely, as an obstruction to stability for almost commuting matrices.

Multiplier algebras are studied in detail, both in the setting of rings and of C^* -algebras. Recent results about extensions of C^* -algebras are discussed, including a result linking amalgamated products with the Busby/Hochschild theory.

ISBN 0-8218-0602-5



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