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Terry A. Loring

American Mathematical Society

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# FIELDS INSTITUTE MONOGRAPHS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

# Lifting Solutions to Perturbing Problems in $C^*$-Algebras

Terry A. Loring

**American Mathematical Society**
Providence, Rhode Island

## The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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ABSTRACT. The theory of $C^*$-algebras determined by generators and relations is developed, with emphasis on the stability properties of those relations. Many lifting and perturbing problems are solved, including problems related to the classification of AD algebras and to almost commuting matrices.

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# Preface

This book began as an attempt to pull together the many papers that comprise the subject called "stable relations for $C^*$-algebras." This is a type of perturbation theory for elements of a $C^*$-algebra. The nature of $C^*$-algebras is such that one cannot study perturbation without also studying the theory of lifting and the theory of extensions.

The approach taken to these subjects is highly algebraic. This is especially true of the preliminary material in part I. Many (apparently) new results are included. Some of these have no good analog in either topology or in the theory of unital rings. I have developed many results in the context of "$\sigma$-unital" rings to make clear that these are essentially algebraic results which happen to only be true, or only be interesting, for rings that behave very much like $C^*$-algebras.

In the event the reader has an adverse reaction to algebra, an introduction is included to explain where all the algebra is to be used.

The reader is assumed to have digested the material on $C^*$-algebras covered in Murphy [1990]. In a few places this is not sufficient, but I have stated the needed result and leave it to the reader to take its veracity on faith or search the designated reference.

I would like to thank those who endured the very early drafts and those who answered many questions or made suggestions: Cristina Cerri, Søren Eilers, Marius Dadarlat, Ken Davidson, Peter Friis, Ken Goodearl, Don Hadwin, Martha Monteiro, Gert Pederson and Jack Spielberg.

Some of the research exposited herein was conducted at the fine facilities of The Fields Institute.

During the past year, the topic of stable relations has become very active. So that this book might appear while still timely, it has been necessary to stay mute on some of the latest results. I will collect recent references to papers on stable relations, after-the-last-minute observations and corrections on my homepage: http://www.math.unm.edu/~loring

Terry A. Loring, April 1996

*With just enough learning to misquote.*
–Lord Byron

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# Bibliography


Abrams, G. D. [1983] *Morita equivalence for rings with local units*, Comm. Algebra **11**, 801–837.

Abrams, G. D., Ánh, P. N. and Mrki, L. [1989] *A topological approach to Morita equivalence for rings with local units*, Rocky Mountain J. Math. **22**, 405–416.

Ackemann, C. A. and Pedersen, G. K. and Tomiyama, J. [1973] *Multipliers of $C^*$-algebra*, J. Funct. Anal. **13**, 277–301.

Akemann, C. A. and Pedersen, G. K. [1977] *Ideal perturbation of elements in $C^*$-algebras*, Math. Scand. **141**, 117–139.

Anderson, J., Blackadar, B. and Haagerup, U. [1991] *Minimal projections in the reduced group $C^*$-algebra of $Z_n * Z_m$*, J. Operator Theory **26**, 3–23.

Ara, P. [1990] *On the symmetric algebra of quotients of a $C^*$-algebra*, Glasgow Math. J. **32**, 377–379.

Ara, P. [1990] *On the symmetric ring of quotients of semiprime rings*, Algebra and geometry (Santiago de Compostela, 1989), Univ. Santiago de Compostela, Santiago de Compostela, pp. 11–15.

Ara, P. [1990] *The extended centroid of $C^*$-algebras*, Archiv Math. **54**, 358–364.

Ara, P. and Goodearl, K. R. [1992] *The almost isomorphism relation for simple regular rings*, Publ. Mat. **36**, 369–388.

Ara, P. and Mathieu, M. [1991] *A local version of the Dauns-Hofmann theorem*, Math. Zeit. **208**, 349–353.

Ara, P. and Mathieu, M. [1993] *An application of local multipliers to centralizing mappings of $C^*$-algebras*, Quart. J. Math. Oxford **44**, 129–138.

Arveson, W. [1977] *Notes on extensions of $C^*$-algebras*, Duke Math. J. **44**, 329-355.

Avitour, D. [1982] *Free Products of $C^*$-algebras*, Trans. Amer. Math. Soc. **271**, 423–435.

Beggs, E. J. and Evans, D. E. [1990] *Real rank zero families of $C^*$-algebras*, La Socit Royale du Canada. L'Academie des Sciences. Comptes Rendus Mathematiques **12**, 179–182.

Beggs, E. J. and Evans, D. E. [1991] *The real rank of algebras of matrix valued functions*, International J. Math. **2**, 131–138.

Berg, I. D. [1974] *On approximation of normal operators by weighted shifts*, Michigan Math. J. **21**, 374–383.

Berg, I. D. [1984] *On operators which almost commute with the shift*, Journal of Operator Theory **11**, 365–377.

Berg, I. D. and Davidson, K. R. [1987] *Almost commuting matrices and the Brown-Douglas-Fillmore theorem*, Bull. Amer. Math. Soc. **16**, 97–100.

Berg, I. D. and Davidson, K. R. [1991] *Almost commuting matrices and a quantitative version of the Brown-Douglas-Fillmore theorem*, Acta. Math. **166**, 121–161.

Bhatia, R. and Holbrook, J. A. R. [1985] *Short normal paths and spectral variation*, Proc. Amer. Math. Soc. **94**, 377–382.

Blackadar, B. [1978] *Weak expectations and nuclear $C^*$-algebras*, Indiana Univ. Math. J. **27**, 1021–1026.

Blackadar, B. [1985] *Shape theory for $C^*$-algebras*, Math. Scand. **56**, 249–275.

Blackadar, B. [1986] *K-theory for Operator Algebras*, M. S. R. I. Monographs No. 5, Springer-Verlag, Berlin and New York.

Blackadar, B. [1990] *Symmetries of the CAR algebra*, Annals of Math. **131**, 589–623.

Blackadar, B. [1993] *Matricial and ultramatricial topology*, Operator algebras, mathematical physics, and low-dimensional topology (Istanbul, 1991), A K Peters, Wellesley, MA, pp. 11–38.

Blackadar, B. [1994] *Projections in $C^*$-algebras*, $C^*$-algebras: 1943–1993 (San Antonio, TX, 1993), Amer. Math. Soc., Providence, RI, pp. 130–149.

Blackadar, B. and Cuntz, J. [1991] *Differential Banach algebra norms and smooth subalgebras of $C^*$-algebras*, J. Operator Theory **26**, 255–282.

Blackadar, B., Bratteli, O. and Elliott, G. A. [1992] *Reduction of real rank in inductive limits of $C^*$-algebras*, Math. Ann. **292**, 111–126.

Blackadar, B., Dadarlat, M. and Rørdam, M. [1991] *The real rank of inductive limit $C^*$-algebras*, Math. Scan. **69**, 211–216.

Blackadar, B., Elliott, G. A. and Kumjian, A. [1992] *Reduction of real rank in inductive limits of $C^*$-algebras*, Math. Annalen **292**, 111–126.

Blackadar, B. and Kirchberg, E., [pnd] *Generalized inductive limits of finite dimensional $C^*$-algebras*, Math. Ann. (to appear).

Blackadar, B., Kumjian, A. and Rørdam, M. [1992] *Approximately central matrix units and the structure of noncommutative tori*, $K$-Theory **6**, 267–284.

Bratteli, O. [1972] *Inductive limits of finite dimensional C*-algebras*, Trans. Amer. Math. Soc. **171**, 195–234.

Bratteli, O. [1993] *The crossed product of the irrational rotation algebra by the flip*, Operator algebras, mathematical physics, and low-dimensional topology (Istanbul, 1991), A K Peters, Wellesley, MA, pp. 61–76.

Bratteli, O. and Elliott, G. A. [1992] *An introduction to fractal $C^*$-algebras*, Operator algebras and topology (Craiova, 1989), Longman Sci. Tech., Harlow, pp. 1–29.

Bratteli, O. and Kishimoto, A. [1992] *Noncommutative spheres. III. Irrational rotations*, Comm. Math. Physics **147**, 605–624.

Bratteli, O., Elliott, G. A. and Kishimoto, A. [1993] *Quasi-product actions of a compact group on a $C^*$-algebra*, J. Funct. Anal. **115**, 313–343.

Bratteli, O., Elliott, G. A., Evans, D. E. and Kishimoto A. [1991] *Finite group actions of AF algebras obtained by folding the interval*, K-theory.

Bratteli, O., Elliott, G. A., Evans, D. E. and Kishimoto, A. [1991a] *Noncommutative spheres. I*, International J. Math. **2**, 139–166.

Bratteli, O., Elliott, G. A., Evans, D. E. and Kishimoto, A. [1991b] *On the classification of inductive limits of inner actions of a compact group*, Current topics in operator algebras (Nara, 1990), World Sci. Publishing, River Edge, NJ, pp. 13–24.

Bratteli, O., Elliott, G. A., Evans, D. E. and Kishimoto, A. [1992] *Noncommutative spheres. II. Rational rotations*, J. Operator Theory. **27**, 53–85.

Bratteli, O., Evans, D. E. and Kishimoto, A. [1993] *Almost shift invariant projections in infinite tensor products*, Quantum and non-commutative analysis (Kyoto, 1992), Kluwer Acad. Publ., Dordrecht, pp. 427–434.

Bratteli, O., Evans, D. E. and Kishimoto, A. [1993a] *Crossed products of totally disconnected spaces by $Z_2 * Z_2$*, Ergodic Theory Dynamical Systems **13**, 445–484.

Bratteli, O., Stormer, E., Kishimoto, A. and Rørdam, M. [1993] *The crossed product of a UHF algebra by a shift*, Ergodic Theory Dynam. Systems **13**, 615–626.

Brown, A., Fong, C. K. and Hadwin, D. W. [1978] *Parts of operators on Hilbert space*, Ill. J. Math. **22**, 306–314.

Brown, L. G. [1976] *Extensions and the structure of $C^*$-algebras*, Symposia Mathematica, Vol. XX (Convegno sulle Algebre $C^*$ e loro Applicazioni, Rome, 1975), Academic Press, London, pp. 539–566.

Brown, L. G. [1982] *Extensions of AF algebras: The projection lifting problem*, Proc. Sympos. Pure Math., vol. 38, American Mathematical Society, Providence, RI.

Brown, L. G. [1983] *The Universal coefficient theorem for Ext and quasidiagonality*, Operator Algebras and Group Representations, vol. 17, Pitman Press, Boston, London and Melbourne, pp. 60–64.

Brown, L. G. [1988] *Determination of $A$ from $M(A)$ and related matters*, La Sociiti Royale du Canada. L'Academie des Sciences. Comptes Rendus Mathematiques **10**, 273–278.

Brown, L. G. [1988a] *Interpolation for multipliers*, Operator algebras and applications, Vol. 1, Cambridge Univ. Press, Cambridge-New York, pp. 55–59.

Brown, L. G. [1988] *Semicontinuity and multipliers of $C^*$-algebras*, Canadian J. of Math. **40**, 865–988.

Brown, L. G. [1991] *Interpolation by projections in $C^*$-algebras of real rank zero*, J. Operator Theory **26**, 383–387.

Brown, L. G. [1992] *Complements to various Stone-Weierstrass theorems for $C^*$-algebras and a theorem of Shultz*, Comm. Math. Phys. vol 143, 405–413.

Brown, L. G. and Pedersen, G. K. [1991] *$C^*$-algebras of real rank zero*, J. Funct. Anal. **99**, 131–149.

Brown, L. G., Douglas, R. G., Fillmore, P. A [1977] Ann. of Math. **105**, 265–324.

Bunce J. and Deddens J. [1975] *A family of simple $C^*$-algebras related to weighted shift operators*, J. Funct. Anal. **19**, 13–24.

Busby, R. C. [1968] *Double centralizers and extensions of $C^*$-algebras*, Trans. Amer. Math. Soc. **132**, 79–99.

Choi, M.-D. [1988] *Almost commuting matrices need not be nearly commuting*, Proc. Amer. Math. Soc. **102**, 528–533.

Choi, M.-D. and Elliott, G. A. [1990] *Density of the selfadjoint elements with finite spectrum in an irrational rotation $C^*$-algebra*, Math. Scand. **67**, 73–86.

Choi, M.-D., [1983] *Lifting projections from quotient $C^*$-algebras*, J. Operator Theory **10**, 21–30.

Chuan, W. F. [1986] *On the generalized functional calculus for nonnormal operators*, J. London Math. Soc. **33**, 499–504.

Combes, F. [1969] *Sur les faces d'une $C^*$-algebre*, Bull. Sci. Math. **93**, 37–62.

Cerri, C. [ppt] *Noncommutative deformations of $C(T^2)$ and $K$-theory*, preprint.

Connes, A. [1994] *Noncommutative Geometry*, Academic Press.

Connes, A. and Cuntz, J. [1988] *Quasi homomorphismes, cohomologie cyclique et positivite*, Comm. Math. Physics **114**, 515–526.

Connes, A. and Higson, N. [1990] *Deformations, morphismes asymptotiques et $K$-theorie bivariante*, C.R. Acad. Sci. Paris **313**, 101–106.

Cuntz, J. [1982] *The $K$-groups for free products of $C^*$-algebras*, Proc. Sympos. Pure Math., vol. 38, American Mathematical Society, Providence, RI.

Cuntz, J. [1987] *A new look at $KK$-theory*, $K$-theory **1**, 31–51.

Cuntz, J. [1991] *Cyclic cohomology and $K$-homology*, Proceedings of the International Congress of Mathematicians (Kyoto, 1990), Math. Soc. Japan, Tokyo, pp. 969–978.

Cuntz, J. [1993] *A survey of some aspects of noncommutative geometry*, Jahr. der Deutschen Mathematiker-Vereinigung **95**, 60–84.

Cuntz, J. [1997] *Simple $C^*$-algebras generated by isometries*, Comm. Math. Phys. **57**, 173–185.

Cuntz, J. and Quillen, D. [1993] *On excision in periodic cyclic cohomology*, C. R. Acad. Sci. Paris **317**, 917–922.

Cuntz, J. and Quillen, D. [1994] *On excision in periodic cyclic cohomology. II. The general case*, C. R. Acad. Sci. Paris **318**, 11–12.

Dadarlat, M. [1988] *Inductive limits of $C^*$-algebras related to some coverings*, Indiana U. Math. J. **37**, 135–143.

Dadarlat, M. [1988a] *On homomorphisms of matrix algebras of continuous functions*, Pacific J. Math. **132**, 227–231.

Dadarlat, M. [1993] *$C^*$-algebras and applications*, Representation theory of groups and algebras, Amer. Math. Soc., Providence, RI, pp. 393–421.

Dadarlat, M. [1993a] *Homotopy after tensoring with uniformly hyperfinite $C^*$-algebras*, $K$-Theory **7**, 133–143.

Dadarlat, M. [1993b] *On the asymptotic homotopy type of inductive limit $C^*$-algebras*, Math. Ann. **297**, 671–676.

Dadarlat, M. [1994c] *A suspension theorem for continuous trace $C^*$-algebras*, Proc. Amer. Math. Soc. **120**, 61–769.

Dadarlat, M. [1994d] *Shape theory and asymptotic morphisms for $C^*$-algebras*, Duke J. **73**, 687–710.

Dadarlat, M. [pnd] *Approximately unitarily equivalent morphisms and inductive limit $C^*$-algebras*, K-theory (to appear).

Dadarlat, M. [ppt] *Reduction to dimension three of local spectra of real rank zero $C^*$-algebras*, preprint.

Dadarlat, M. and Gong, G. [ppt] *A classification result for approximately homogeneous $C^*$-algebras of real rank zero*, preprint.

Dadarlat, M. and Loring, T. A. [1992] *The $K$-theory of abelian subalgebras of $AF$ algebras*, J. reine angew. Math. **432**, 39-55.

Dadarlat, M. and Loring, T. A. [1994] *Extensions of certain real rank zero $C^*$-algebras*, Ann. Inst. Fourier **44**, 907–925.

Dadarlat, M. and Loring, T. A. [1994a] *K-homology, asymptotic representations and unsuspended E-theory*, J. Funct. Anal. **126**, 367–383.

Dadarlat, M. and Loring, T. A. [1994b] *Deformations of topological spaces predicted by E-theory*, Algebraic Methods in Operator Theory, Birkhauser, Basel.

Dadarlat, M. and Loring, T. A. [ppt] *Classifying $C^*$-algebras via ordered, mod-p K-theory*, preprint.

Dadarlat, M. and Nemethi A. [1990] *Shape Theory and (connective) K-theory*, J. Operator Theory **23**, 207–291.

Dadarlat, M. and Pasnicu, C. [1989] *Inductive limits of $C(X)$-modules and continuous fields of AF-algebras*, Journal of Functional Analysis **85**, 103–116.

Dadarlat, M., Nagy, G., Nemethi, A. and Pasnicu, C. [1992] *Reduction of topological stable rank in inductive limits of $C^*$-algebras.*, Pacific J. Math. **153**, 267–276.

Dadarlat, M., Nagy, G. and Nemethi, A. [1992] *Reduction of topological stable rank in inductive limits of $C^*$-algebras*, Pacific J. Math. **153**, 267–276.

Davidson, K. R. [1985] *Almost commuting Hermitian matrices*, Math. Scand. **56**, 222–240.

Davidson, K. R. [1991] *Lifting positive elements in $C^*$-algebras*, Integral Eq. and Operator Theory **14**, 183–191.

Davidson, K. R. and Huang, H. [1994] *Universal factorization of positive operators*, Indiana Univ. Math. J. **43**, 131–142.

Davidson, K. R., Ordower, M. S. [1994] *Some exact distance constants*, Linear Algebra Appl. **208/209**, 37–55.

Davidson, K. R. [ppt] *$C^*$-Algebras by Example.*

Dykema, K., Nica, A. and Voiculescu, D. [1992] *Free Random Variables,*, Amer. Math. Soc., Providence, RI.

Effros, E. G. [1982] *On the structure of $C^*$-algebras. Some old and new problems*, Operator algebras and applications, Proc. Symp. Pure Math. no. 48 part 1, Amer. Math. Soc., Providence RI, pp. 19–34.

Effros, E. G. [1985] *Lifting problems and local reflexivity for $C^*$-algebras*, Duke Mathematical Journal **52**, 103.

Effros, E. G. [1991] *Dimensions and $C^*$-algebras*, CBMS Regional Conf. Series in Math., vol. 46, Amer. Math. Soc., Providence R. I..

Effros, E. G. and Kaminker J. [1985] *Homotopy continuity and shape theory for $C^*$-algebras*, Geometric methods in operator algebras, U.S.-Japan seminar at Kyoto 1983, Pitman 1985.

Effros, E. G. and Kaminker J. [1986] *Some homotopy and shape computations*, Group Representations, Ergodic Theory, Operator Algebras and Mathematical Physics (C. C. Moore, ed.), MSRI series no. 6 1986, Springer-Verlag.

Eilers, S., Loring, T. A. and Pedersen, G. K. [ppt] *Morphisms of extensions of $C^*$-algebras: pushing forward the Busby invariant*, preprint, Adv. Math., to appear.

Elliott, G. A. [1976] *On the classification of inductive limits of sequences of semisimple finite-dimensional algebras*, J. Algebra **39**, 29–44.

Elliott, G. A. [1987] *The ideal structure of the multiplier algebra of an AF algebra*, La Socit Royale du Canada. L'Academie des Sciences. Comptes Rendus Mathmatiques **9**, 225–230.

Elliott, G. A. [1993] *On the classification of $C^*$-algebras of real rank zero I*, J. reine angew. Math. **443**, 179-219.

Elliott, G. A. [1994] *Normal elements of a simple $C^*$-algebra*, Algebraic methods in operator theory, Birkhäuser, Boston, MA, pp. 109–123.

Elliott, G. A. and Evans, D. E. [1993] *The structure of the irrational rotation $C^*$-algebra*, Annals Math. **138**, 477–501.

Elliott, G. A. and Gong, G. [ppt] *On the classification of $C^*$-algebras of real rank zero II*, preprint.

Elliott, G. A., Gong, G. and Lin, H. X. [1994] *Homomorphisms, homotopies, and approximations by circle algebras*, C. R. Math. Rep. Acad. Sci. Canada **16**, 45–50.

Elliott, G. A., Gong, G., Lin, H. X. and Pasnicu, C. [1994] *Homomorphisms, homotopies, and approximations by circle algebras*, C. R. Math. Rep. Acad. Sci. Canada **16**, 45–50.

Elliott, G. A. and Rørdam, M. [1993] *The automorphism group of the irrational rotation $C^*$-algebra*, Communications in Mathematical Physics **155**, 3–26.

Elliott, G. A., Exel, R. and Loring, T. A. [1991] *The soft torus III: the flip*, J. Operator Theory **26**, 333–344.

Elliott, G. A. and Loring, T. A. [1992] *AF embeddings of $C(\mathbb{T}^2)$ with prescribed K-theory*, J. Funct. Anal. **103**, 1–25.

Evans, D. E. [1992] *Some noncommutative orbifolds*, Ideas and methods in mathematical analysis, stochastics, and applications (Oslo, 1988), Cambridge Univ. Press, Cambridge, pp. 344–364.

Evans, D. E. and Kishimoto, A. [1991] *Compact group actions on AF algebras obtained by folding the interval*, $K$-Theory **xx**, xxx–xxx.

Evans, D. E. and Kishimoto, A. [1991] *Compact group actions on UHF algebras obtained by folding the interval*, J. Funct. Anal. **98**, 346–360.

Exel, R. [1993] *The soft torus and applications to almost commuting matrices*, Pacific J. Math. **160**, 207–217.

Exel, R. [1994] *The soft torus: a variational analysis for commutator norms*, J. Funct. Anal. **126**, 259–273.

Exel, R. and Loring, T. A. [1992] *Finite-dimensional representations of free product $C^*$-algebras*, Internat. J. Math. **3**, 464–476.

Farsi, C. [ppt] preprint.

Friis, P. and Rørdam, M. [ppt] *Almost commuting self-adjoint matrices–a short proof of Huaxin Lin's theorem*, preprint.

Glimm, J. [1960] *On a certain class of operator algebras*, Trans. Amer. Math. Soc. **95**, 318–340.

Glimm, J. [1961] *Type I $C^*$-algebras*, Annals of Math. **73**, 572–612.

Glimm, J. and Kadison, R. V. [1960] *Unitary operators in $C^*$-algebras*, Pacific J. Math. **10**, 547–556.

Gong, G. [1994] *Approximation by dimension drop $C^*$-algebras and classification*, C. R. Math. Rep. Acad. Sci. Canada **16**, 40–44.

Gong, G. [ppt] *Classification of $C^*$-algebras of real rank zero and unsuspended E-equivalent types*, preprint.

Gong, G. and Lin, H. X. [1992] *The exponential rank of inductive limit $C^*$-algebras*, Math. Scand. **71**, 301–319.

Goodearl, K. R. and Menal, P. [1990] *Free and residually finite-dimensional $C^*$-algebras*, J. Funct. Anal. **90**, 391–410.

Grove, K. and Pedersen, G. K. [1984a] *Diagonalizing matrices over $C(X)$*, J. Funct. Anal. **59**, 65–89.

Grove, K. and Pedersen, G. K. [1984b] *Sub-Stonean spaces and corona sets*, J. Funct. Anal. **56**, 124–143.

Hadwin, D. W. [1978] *Continuous functions of operators; a functional calculus*, Indiana Univ. Math. J. **27**, 113–125.

Hadwin, D. W. [1978a] *An asymptotic double commutant theorem*, Trans. Amer. Math. Soc. **244**, 273–297.

Hadwin, D. W. [1994] *Closures of direct sums of classes of operators*, Proceedings of the American Mathematical Society **121**, 697–701.

Hadwin D. W. [ppt] *Lifting algebraic elements in $C^*$-algebras*, preprint.

Hadwin, D. W., Kaonga L. and Mathes, B. [ppt] *Noncommutative continuous functions*, preprint.

Hadwin, D. W. and Loring, T. A. [ppt] *Normal operators in $C^*$-algebras without nice approximants*, preprint.

Higson, N. [1987] *On a technical theorem of Kasparov*, J. Funct. Anal. **73**, 107–112.

Higson, N. [1988] *Algebraic $K$-theory of stable $C^*$-algebras*, Advances in Math. **67**.

Higson, N. [1990] *An approach to $\mathbf{Z}/k$-index theory*, International J. Math. **1**, 189–210.

Hochshild, G. [1947] *Cohomology and representations of associative algebras*, Duke Math. J. **14**, 921–948.

Johnson, B. E. [1972] *Cohomology in Banach algebras*, Mem. Amer. Math. Soc . **127**.

Kadison, R. V. [1957] *Irreducible operator algebras*, Proc. Natl. Acad. Sci. USA **43**, 273–276.

Kasparov, G. G. [1980] *The operator $K$-functor and extensions of $C^*$-algebras*, Izv. Akad. Nauk. SSSR **44**, 571–636.

Kasparov, G. G. [1981] *$K$-theory, group $C^*$-algebras and higher signatures (conspectus)*, preprint, revised as [Kasparov-1988].

Kasparov, G. G. [1984] *Operator $K$-theory and its applications: elliptic operators, group representations, higher signatures, $C^*$-extensions*, Proceedings of the International Congress of Mathematicians, PWN, Warsaw, pp. 987–1000.

Kasparov, G. G. [1988] *Equivariant $KK$-theory and the Novikov conjecture*, Inven. Math. **91**, 147–201.

Kato, T. [1966] *Perturbation Theory for Linear Operators*, Springer Verlag, New York.

Kishimoto, A. [1990] *Actions of finite groups on certain inductive limit C*-algebras*, International J. Math. **1**, 267–292.

Komatsu, H. [1986] *The category of s-unital modules*, Math. J. Okayama University **28**, 65–91.

Komatsu, H. and Tominaga, H. [1985] *On s′-unital, s″-unital, and s*-unital rings*, Proceedings of the 8th symposium on semigroups (Matsue, 1984), Shimane Univ., Matsue, pp. 45–51.

Komatsu, H. and Tominaga, H. [1986] *On commutativity of s-unital rings*, Math. J. Okayama University **28**, 93–96.

Komatsu, H. and Tominaga, H. [1989] *Some commutativity theorems for left s-unital rings*, Resultate Math. **15**, 335–342.

Lin, H. X. [1988] *Ideals of multiplier algebras of simple AF C*-algebras*, Proc. Amer. Math. Soc. **104**, 239–244.

Lin, H. X. [1989] *The simplicity of the quotient algebra $M(A)/A$ of a simple C*-algebra*, Math. Scand. **65**, 119–128.

Lin, H. X. [1992] *Approximation by normal elements with finite spectrum in C*-algebras of real rank zero*, Preprint.

Lin, H. X. [1993b] *Exponential rank of C*-algebras with real rank zero and the Brown-Pedersen conjectures*, J. Funct. Anal. **114**, 1–11.

Lin, H. X. [1993b] *Extensions by a C*-algebras with real rank zero*, Internat. J. Math. **4**, 231–252.

Lin, H. X. [1994] *Almost normal elements*, Internat. J. Math. **5**, 765–778.

Lin, H. X. [ppt] *Almost commuting selfadjoint matrices and applications*, preprint.

Lin, H. X. [ppt-a] *Homomorphisms from $C(X)$ into C*-algebras*, preprint.

Lin, H. X. and Rørdam, M. [1995] *Extensions of inductive limits of circle algebras*, J. London Math. Soc., **51**, 603–613.

Lin, H. X. and Zhuang, S. [1992] *Certain simple C*-algebras with nonzero real rank whose corona algebras have real rank zero*, Houston Journal of Mathematics **18**, 57–71.

Loring, T. A. [1988] *K-theory and almost commuting matrices*, Canad. J. Math. **40**, 197-216.

Loring, T. A. [1991] *The K-theory of AF embeddings of the rational rotation algebras*, $K$-theory **4**, 227-243.

Loring, T. A. [1991a] *Berg's technique for pseudo-actions with applications to AF embeddings*, Canad. J. Math. **43**, 119–157.

Loring, T. A. [1993] *C*-algebras generated by stable relations*, J. Funct. Anal. **112**, 159-201.

Loring, T. A. [1993a] *Projective C*-algebras*, Math. Scand. **73**, 274–280.

Loring, T. A. [1995] *Normal elements of C*-algebras without finite-spectrum approximants*, J. London Math. Soc. **51**, 353–364.

Loring, T. A. [pnd] *Stable relations II: corona semiprojectivity and dimension-drop C*-algebras*, Pacific J. Math. (to appear).

Loring, T. A. [pnd-a] *Perturbation questions in the Cuntz picture of K-theory*, $K$-Theory (to appear).

Loring, T. A. [ppt] *When matrices commute*, preprint.

Loring, T. A. and Pedersen, G. K. [pnd] *Corona extendibility and asymptotic multiplicativity*, $K$-theory (to appear).

Loring, T. A. and Pedersen, G. K. [ppt] *projectivity, transitivity and AF telescopes*, preprint.

Loring, T. A. and Pedersen, G. K. [ppt-b] *Smoothing techniques in C*-algebra theory*, preprint.

Loring, T. A. and Spielberg, J. S. [1994] *Approximation of normal elements in the multiplier algebra of an AF C*-algebra*, Proc. Amer. Math. Soc. **121**, 1173–1175.

Mardesic and Segal, J. [1982] *Shape Theory*, North-Holland, Amsterdam.

McClanahan, K. [1992] *C*-algebras generated by elements of a unitary matrix*, J. Funct. Anal. **107**, 439–457.

McClanahan, K. [1993] *K-theory and Ext-theory for rectangular unitary C*-algebras*, Rocky Mountain J. Math. **23**, 1063–1080.

McClanahan, K. [1994] *Simplicity of reduced amalgamated products of C*-algebras*, Canad. J. Math. **46**, 793–807.

Mingo, J. A. and Spielberg, J. S. [1991] *The index of normal Fredholm elements of C*-algebras*, Proc. Amer. Math. Soc. **113**, 187–192.

Mishchenko, A. S. [pnd] *Almost Fredholm representations of discrete groups* (to appear).

Murphy, G. J. [1988] *Diagonality in C*-algebras*, Math. Zeit. **199**, 279–284.

Murphy, G. J. [1990] *C*-algebras and operator theory*, Academic Press, New York.

Olsen, C. L. [1971] *A structure theorem for polynomially compact operators*, Amer. J. Math. **93**, 686–698.

Olsen, C. L. and Pedersen, G. K. [1989] *Corona $C^*$-algebras and their applications to lifting problems*, Math. Scand. **64**, 63-86.

Pasnicu, C. [1988] *On inductive limits of certain $C^*$-algebras of the form $C(X) \otimes F$*, Trans. Amer. Math. Soc. **310**, 703–714.

Pasnicu, C. [1988] *Study of some special classes of operator algebras*, Stud. Cer. Mat. **40**, 175–231.

Pasnicu, C. [1992] *Homomorphisms of Bunce-Deddens algebras*, Pacific J. Math. **155**, 157–167.

Pasnicu, C. [1992a] *Homomorphisms compatible with covering maps of the two-torus*, C. R. Math. Rep. Acad. Sci. Canada **14**, 143–147.

Pasnicu, C. [1992b] *Homomorphisms compatible with some covering maps*, C. R. Math. Rep. Acad. Sci. Canada **14**, 263–267.

Passman, D. S. [1987] *Computing the symmetric ring of quotients*, J. Algebra **105**, 207–235.

Pedersen, G. K. [1979] *$C^*$-algebras and their automorphism groups*, Academic Press, New York.

Pedersen, G. K. [1984] *Multipliers of $AW^*$-algebras*, Math. Zeit. **187**, 23–24.

Pedersen, G. K. [1986] *$SAW^*$-algebras and corona $C^*$-algebras, contributions to noncommutative topology*, Jour. Operator Th. **15**, 15–32.

Pedersen, G. K., [1990] *The corona construction*, Proc. of the 1988 GPOTS-Wabash Conf. (J. B. Conway and B. B. Morrel, ed.), Longman Sci. & Tech., Harlow, pp. 49–92.

Pedersen, G. K. [pnd] *A strict version of the noncommutative Urysohn Lemma*, Proc. Amer. Math. Soc. (to appear).

Phillips, N. C. [1988] *A new approach to the multipliers of Pedersen's ideal*, Proc. Amer. Math. Soc. **104**, 861–867.

Phillips, N. C. [1991] *Simple $C^*$-algebras with the property weak (FU)*, Math. Scand. **69**, 127–151.

Phillips, N. C. [1988] *Inverse limit of $C^*$-algebras and applications*, Operator Algebras and Applications, London Math. Soc.

Power, S. C. [1991] *Nonselfadjoint operator algebras and inverse systems of simplicial complexes*, J. Reine Angew. Math. **421**, 43–61.

Power, S. C. [1992] *Limit algebras: an introduction to subalgebras of $C^*$-algebras Pitman Research Notes in Mathematics Series, 278*, Longman Scientific &Technical, Harlow.

Power, S. C. [1992a] *Algebraic orders on $K_0$ and approximately finite operator algebras*, J. Operator Th. **27**, 87–106.

Quigg, J. C. and Spielberg, J. S. [1992] *Regularity and hyporegularity in $C^*$-dynamical systems*, J. Math. **18**, 139–152.

Rosenberg, J. and Schochet, C. [1987] *The Künneth theorem and the universal coefficient theorem for Kasparov's generalized functor*, Duke Math. J. **55**, 431–474.

Rowen, L. H. [1991] *Ring theory (student edition)*, Academic Press, New York.

Schochet, C. [1984] *Topological methods for $C^*$-algebras IV*, Pacific J. Math. **114**, 447–468.

Schweitzer, L. B. [1992] *A short proof that $M_n(A)$ is local if $A$ is local and Frechet*, Internat. J. Math. **3**, 581–589.

Smith, R. R. and Williams, D. P. [1986] *The decomposition property for $C^*$-algebras*, J. Operator Theory **16**, 41–74.

Smith, R. R. and Williams, D. P. [1988] *Separable injectivity for $C^*$-algebras*, Indiana Univ. Math. J. **37**, 111–133.

Spielberg, J. S. [1991] *Free-product groups, Cuntz-Krieger algebras, and covariant maps*, Internat. J. Math. **2**, 457–476.

Spielberg, J. S. [1991a] *Type III factor states on $O_2$ which extend the trace on Choi's algebra*, J. Operator Theory **25**, 215–236.

Spielberg, J. S. [1993] *Cuntz-Krieger algebras associated with Fuchsian groups*, Ergodic Theory Dynamical Systems **13**, 81–595.

Stratila, A. and Voiculescu, D. [1975] *Representations of AF-Algebras and of the group $U(\infty)$, Lecture Notes in Mathematics no. 486*, Springer-Verlag, Berlin.

Voiculescu, D. [1981] *Remarks on the singular extension of the Heisenberg group*, J. Operator Theory **5**, 147–170.

Voiculescu, D. [1983] *Asymptotically commuting finite rank unitary operator without commuting approximants*, Acta. Sci. Math. **45**, 429–431.

Wegge-Olsen, N. E. [1993] *K-theory and $C^*$-algebras*, Oxford University Press, New York.

Zhang, S. [1989] *On the structure of projections and ideals of corona algebras*, Canadian J. Math. **41**, 721–742.

Zhang, S. [1990] *A Riesz decomposition property and ideal structure of multiplier algebras*, J. Operator Th. **24**, 209–225.

Zhang, S. [1990a] *$C^*$-algebras with real rank zero and the internal structure of their corona and multiplier algebras. III*, Canad. J. Math. **42**, 159–190.

Zhang, S. [1991] *$K_1$-groups, quasidiagonality and interpolation by multiplier projections*, Trans. Amer. Math. Soc. **325**, 793–818.

Zhang, S. [1992] *Certain $C^*$-algebras with real rank zero and their corona and multiplier algebras. I*, Pacific J. Math. **155**, 169–197.

Zhang, S. [1992a] *$C^*$-algebras with real rank zero and their corona and multiplier algebras. IV*, Internat. J. Math. **3**, 309–330.

Zhang, S. [1992b] *Certain $C^*$-algebras with real rank zero and their corona and multiplier algebras. II*, $K$-Theory **6**, 1–27.

# Index

**Lifting Solutions to Perturbing Problems in C*-Algebras**
Terry A. Loring

The nature of $C^*$-algebras is such that one cannot study perturbation without also studying the theory of lifting and the theory of extensions. Approximation questions involving representations of relations in matrices and $C^*$-algebras are the central focus of this volume. A variety of approximation techniques are unified by translating them into lifting problems: from classical questions about transitivity of algebras of operators on Hilbert spaces to recent results in linear algebra. One chapter is devoted to Lin's theorem on approximating almost normal matrices by normal matrices.

The techniques of universal algebra are applied to the category of $C^*$-algebras. An important difference, central to this book, is that one can consider approximate representations of relations and approximately commuting diagrams. Moreover, the highly algebraic approach does not exclude applications to very geometric $C^*$-algebras.

$K$-theory is avoided, but universal properties and stability properties of specific $C^*$-algebras that have applications to $K$-theory are considered. Index theory arises naturally, and very concretely, as an obstruction to stability for almost commuting matrices.

Multiplier algebras are studied in detail, both in the setting of rings and of $C^*$-algebras. Recent results about extensions of $C^*$-algebras are discussed, including a result linking amalgamated products with the Busby/Hochshild theory.

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