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B. V. Rajarama Bhat
George A. Elliott
Peter A. Fillmore
Editors

American Mathematical Society

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The Fields Institute
for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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Preface

The articles in this volume are based for the most part on lectures given at The Fields Institute for Research in Mathematical Sciences by participants in the program, "Operator Algebras and Applications", held during the year 1994–1995 in Waterloo, Ontario. (It is the ninth volume related to the proceedings of this program.)

The scientific organizing committee for this program consisted of Alain Connes, Man-Duen Choi, Kenneth R. Davidson, George A. Elliott (chairman), Peter A. Fillmore, David E. Handelman, Nigel Higson, Vaughan F. R. Jones, Ian F. Putnam, and Dan-Virgil Voiculescu.

In order of appearance (in this volume), the lectures (together with the corresponding note-takers, mentioned in parentheses) were Peter Fillmore (three lectures: Douglas Harder, Peter Friis, Jakob Mortensen), Christopher Phillips (Jonathan Samuel), Mikael Rørdam (Jonathan Samuel), Berndt Brenken (David Kerr), Alexander Kumjian (Peter Friis), Kenneth Dykema (Teresa Bates), Jerome Kaminker (Richard Gjerde), Man-Duen Choi (Andrew Dean), Kenneth Davidson (Douglas Harder), Derek Robinson (Teresa Bates), Noberto Salinas, Alexandru Nica (David Kerr), Gabriel Nagy (Thomas Gauguin Houghton-Larsen), Florin Boca (two lectures: Kevin Fitzgerald and Walter Schreiner, Claus Dahl), James Mingo (Thomas McLeister), Florin Boca (Claus Dahl), Man-Duen Choi (Espen Husstad), Kenneth Dykema (Kenneth Stevens), William Arveson (Rajarama Bhat and Ileana Ionescu), Ola Bratteli (Jonathan Samuel), George Elliott (two lectures: Massoud Amini), Jesper Villadsen (Andrew Dean), Mikael Rørdam (Peter Friis), Irina Stevens (Ph.D. Thesis), Adrian Ocneanu (special lecture series: Satoshi Goto).

The editors trust that the record of these lectures will be pertinent, at least in some degree, to all future investigations into the subject.

B. V. Rajarama Bhat
George A. Elliott
Peter A. Fillmore

Toronto, August 17, 1999

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Bibliography

[1] Cappelli, A., Itzykson, C. and Zuber, J.-B. [1987], *The A-D-E classification of minimal and $A_1^{(1)}$ conformal invariant theories*, Comm. Math. Phys. **113**, 1–26.

[2] Drinfel'd, V. G., *Quantum groups*, Proc. ICM-86, Berkeley, 798–820.

[3] Evans, D. E. and Kawahigashi, Y. [1994], *The E_7 commuting squares produce D_{10} as principal graph*, Publ. RIMS. Kyoto Univ. **30**, 151–166.

[4] Goodman, F., de la Harpe, P. and Jones, V. F. R. [1989], *Coxeter graphs and towers of algebras*, MSRI publications 14, Springer.

[5] Jones, V. F. R. [1983], *Index for subfactors*, Invent. Math. **72**, 1–15.

[6] Jones, V. F. R. [1985], *A polynomial invariant for knots via von Neumann algebras*, Bull. Amer. Math. Soc. **12**, 103–112.

[7] Kauffman, L. H. and Lins, S. [1995], *Temperley-Lieb recoupling theory and invariants of 3-manifolds*, Ann. Math. Studies **133**, Princeton University Press.

[8] Kawahigashi, Y. [1995], *Classification of paragroup actions on subfactors*, Publ. RIMS, Kyoto Univ **31**, 481–517.

[9] Ocneanu, A. [1988], *Quantized group string algebras and Galois theory for algebras*, in Operator algebras and applications, Vol. 2 (Warwick, 1987), London Math. Soc. Lect. Note Series Vol. 136, Cambridge University Press, pp. 119–172.

[10] Ocneanu, A. [1989], *Graph geometry, quantized groups and nonamenable subfactors*, Lake Tahoe Lectures, June–July.

[11] Ocneanu, A. [1991], *Quantum symmetry, differential geometry of finite graphs and classification of subfactors*, University of Tokyo Seminary Notes 45, (Notes recorded by Y. Kawahigashi).

[12] Ocneanu, A. [1991], *An invariant coupling between 3-manifolds and subfactors, with connections to topological and conformal quantum field theory*, preprint.

[13] Ocneanu, A. [1991], *Operator algebras, 3-manifolds and quantum field theory*, OHP sheets for the Istanbul talk.

[14] Ocneanu, A. [1991], Lectures at Collège de France.

[15] Ocneanu, A. [1993], Seminar talk at University of California, Berkeley.

[16] Ocneanu, A. [1993], *Chirality for operator algebras*, (recorded by Y. Kawahigashi), Subfactors — Proceedings of the Taniguchi Symposium, Subfactors (Kyuzseo, 1993) World Sci. Publ., Singapore, 1994, pp. 39–63.

[17] Okamoto, S. [1991], *Invariants for subfactors arising from Coxeter graphs*, in Current Topics in Operator Algebras, World Scientific Publishing, pp. 84–103.

[18] Reshetikhin, N. Yu. and Turaev, V. G. [1991], *Invariants of 3-manifolds via link polynomials and quantum groups*, Invent. Math. **103**, 547–597.

[19] Slodowy, P. [1990], *A new A-D-E classification (after A. Capelli, C. Itzykson, J.-B. Zuber)*, Bayreuther Math. Sch. **33**, 197–213.

[20] Turaev, V. G. and Viro, O. Y. [1992], *State sum invariants of 3-manifolds and quantum 6j-symbols*, Topology, **31**, 865–902.

[21] Verlinde, E. [1988], *Fusion rules and modular transformation in 2D conformal field theory*, Nucl. Phys. **B300**, 360–376.

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