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Lectures on Algebraic Model Theory

Bradd Hart
Matthew Valeriote
Editors



American Mathematical Society

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The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world’s most prestigious prize for mathematics research—the Fields Medal.

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Preface

During the academic year 1996/97 the Fields Institute for Research in Mathematical Sciences held a year-long programme in Algebraic Model Theory. During the first semester of the programme three lecture series were presented which covered recent developments in Differential Fields, o-Minimality, and Finite Algebra. This volume contains the lecture notes from those series.

The editors wish to thank those students who kindly assisted in the preparation of these notes.

Bradd Hart
Matthew Valeriote
Editors

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Bradd Hart and Matthew Valeriote, Editors

In recent years, model theory has had remarkable success in solving important problems as well as in shedding new light on our understanding of them. The three lectures collected here present recent developments in three such areas: Anand Pillay on differential fields, Patrick Speissegger on o-minimality and Matthias Clasen and Matthew Valeriote on tame congruence theory.

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