Lectures on Quantum Groups

Jens Carsten Jantzen

Graduate Studies in Mathematics
Volume 6

American Mathematical Society

http://dx.doi.org/10.1090/gsm/006

Other Titles in This Series

6 **Jens Carsten Jantzen,** Lectures on quantum groups, 1996
5 **Rick Miranda,** Algebraic curves and Riemann surfaces, 1995
4 **Russell A. Gordon,** The integrals of Lebesgue, Denjoy, Perron, and Henstock, 1994
3 **William W. Adams and Philippe Loustaunau,** An introduction to Gröbner bases, 1994
2 **Jack Graver, Brigitte Servatius, and Herman Servatius,** Combinatorial rigidity, 1993
1 **Ethan Akin,** The general topology of dynamical systems, 1993

Lectures on Quantum Groups

Lectures on Quantum Groups

Jens Carsten Jantzen

Graduate Studies
in Mathematics
Volume 6

American Mathematical Society

2000 *Mathematics Subject Classification.* Primary 17B37.

ABSTRACT. This book is an introduction to the theory of quantum groups. Its main objects are the quantized enveloping algebras introduced independently by Drinfeld and Jimbo. We study their finite dimensional representations, their centers, and their bases. In particular, we look at the crystal (or canonical) bases discovered independently by Lusztig and Kashiwara.

We first look at the quantum analogue of the Lie algebra $\mathfrak{sl}_2$, and then at the quantum analogue of arbitrary finite dimensional complex Lie algebras. The book is directed to anyone who wants to learn the subject and has been introduced to the theory of finite dimensional complex Lie algebras.

Library of Congress Cataloging-in-Publication Data

Jantzen, Jens Carsten.
Lectures on quantum groups / Jens Carsten Jantzen.
p. cm. — (Graduate studies in mathematics, ISSN 1065-7339; v. 6)
Includes bibliographical references and index.
ISBN 0-8218-0478-2 (alk. paper)
1. Quantum groups. 2. Mathematical physics. I. Title. II. Series.

QC20.7.G70J36 1995 95-25393
512′.55—dc20 CIP

♾ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 14 13 12 11 10 09

Contents

References

I refer to the books and papers by the names of the author(s) plus (where appropriate) a number. There are two exceptions: [H] = [Humphreys 1] and [L] = [Lusztig 7].

E. Abe
Hopf Algebras (Cambridge Tracts in Mathematics), Cambridge etc. 1980 (Cambridge Univ.)

H. H. Andersen, P. Polo, Wen K.
Representations of quantum algebras, *Invent. math.* **104** (1991), 1–59

N. Bourbaki
1) *Algèbre, Chap. 2,* Paris (3e éd.) 1967 (Hermann)
2) *Groupes et algèbres de Lie, Chap. 4, 5 et 6,* Paris 1968 (Hermann)
3) *Groupes et algèbres de Lie, Chap. 7 et 8,* Paris 1975 (Hermann)

V. Chari, A. N. Pressley
A Guide to Quantum Groups, Cambridge etc. 1994 (Cambridge Univ.)

C. De Concini, C. Procesi
Quantum Groups, pp. 31–140 in: L. Boutet de Monvel et al., *D–modules, Representation Theory, and Quantum Groups,* Proc. Venezia 1992 (Lecture Notes in Mathematics **1565**), Berlin etc. 1993 (Springer)

C. De Concini, V. G. Kac
Representations of quantum groups at roots of 1, pp. 471–506 in: A. Connes et al. (eds.), *Operator Algebras, Unitary Representations, Enveloping Algebras, and Invariant Theory* (Colloque Dixmier), Proc. Paris 1989 (Progress in Mathematics **92**), Boston etc. 1990 (Birkhäuser)

C. De Concini, V. G. Kac, C. Procesi
Some quantum analogues of solvable Lie groups, preprint

P. Deligne, J. S. Milne
Tannakian categories, pp. 101–228 in: P. Deligne, J. S. Milne, A. Ogus, K.-y. Shih: *Hodge Cycles, Motives, and Shimura Varieties* (Lecture Notes in Mathematics **900**), Berlin etc. 1981 (Springer)

M. Demazure, P. Gabriel
Groupes Algébriques, Tome I, Paris & Amsterdam 1970 (Masson & North–Holland)

J. Dixmier

Algèbres Enveloppantes, Paris etc. 1974 (Gauthier-Villars)

V. G. Drinfel'd

1) Hopf algebras and the quantum Yang-Baxter equation, *Soviet Math. Doklady* **32** (1985), 254–258, (russ. orig.:) Алгебры Хопфа и квантовое уравнение Янга-Бакстера, Докл. Акад. Наук СССР **283** (1985), 1060–1064
2) Quantum groups, pp. 798–820 in: A. M. Gleason (ed.), *Proceedings of the International Congress of Mathematicians 1986, vol. 1,* Proc. Berkeley 1986, [Providence, R. I., 1987 (Amer. Math. Soc.)]

J. E. Humphreys

1) *Introduction to Lie Algebras and Representation Theory* (Graduate Texts in Mathematics **9**), New York etc. (3rd printing) 1980 (Springer)
2) *Reflection Groups and Coxeter Groups* (Cambridge Studies in Advanced Mathematics **29**), Cambridge 1990 (Cambridge Univ.)

N. Jacobson

Lie Algebras (Interscience Tracts in Pure and Applied Mathematics **10**), New York etc. (3rd printing) 1966 (Interscience/Wiley)

M. Jimbo

1) A q–difference analogue of $U(\mathfrak{g})$ and the Yang-Baxter equation, *Lett. Math. Phys.* **10** (1985), 63–69
2) A q–difference analogue of $U(\mathfrak{gl}(N+1))$, Hecke algebra and the Yang-Baxter equation, *Lett. Math. Phys.* **11** (1986), 247–252

A. Joseph

Quantum Groups and Their Primitive Ideals, Ergebnisse der Mathematik (3) **29**, Berlin etc. 1995 (Springer)

A. Joseph, G. Letzter

1) Local finiteness of the adjoint action for quantized enveloping algebras, *J. Algebra* **153** (1992), 289–318
2) Rosso's form and quantized Kac-Moody algebras, preprint

M. Kashiwara

1) Crystalizing the q–analogue of universal enveloping algebras, *Comm. Math. Phys.* **133** (1990), 249–260
2) On crystal bases of the q–analogue of universal enveloping algebras, *Duke Math. J.* **63** (1991), 465–516
3) The crystal base and Littelmann's refined Demazure character formula, *Duke Math. J.* **71** (1993), 839–858

M. Kashiwara, T. Nakashima

Crystal graphs for representations of the q–analogue of classical Lie algebras, *J. Algebra* **165** (1994), 295–345

C. Kassel

Quantum Groups, Graduate Texts in Mathematics **155**, New York etc. 1995 (Springer)

A. N. Kirillov, N. Reshetikhin

q–Weyl group and a multiplicative formula for universal R–matrices, *Comm. Math. Phys.* **134** (1990), 421–431

S. Z. Levendorskiĭ, Ya. S. Soibelman
Some applications of the quantum Weyl groups, *J. Geom. Phys.* **7** (1990), 241–254

G. Lusztig
1) Quantum deformations of certain simple modules over enveloping algebras, *Adv. in Math.* **70** (1988), 237–249
2) Finite dimensional Hopf algebras arising from quantized universal enveloping algebras, *J. Amer. Math. Soc.* **3** (1990), 257–296
3) On quantum groups, *J. Algebra* **131** (1990), 466–475
4) Quantum groups at roots of 1, *Geom. Dedicata* **35** (1990), 89–114
5) Canonical bases arising from quantized enveloping algebras, *J. Amer. Math. Soc.* **3** (1990), 447–498
6) Quivers, perverse sheaves, and quantized enveloping algebras, *J. Amer. Math. Soc.* **4** (1991), 365–421
7) *Introduction to Quantum Groups* (Progress in Mathematics **110**), Boston etc., 1993 (Birkhäuser)

I. G. Macdonald
Symmetric Functions and Hall Polynomials, Oxford 1979 (Clarendon, Oxford Univ.)

Yu. I. Manin
Quantum Groups and Non-commutative Geometry, Montréal 1988 (CRM, Univ. de Montréal)

T. Nakashima
Crystal base and a generalization of the Littlewood-Richardson rule for the classical Lie algebras, *Comm. Math. Phys.* **154** (1993), 215–243

M. Rosso
Analogues de la forme de Killing et du théorème d'Harish-Chandra pour les groupes quantiques, *Ann. scient. Éc. Norm. Sup.* (4) **23** (1990), 445–467

A. N. Rudakov, I. R. Shafarevich
Irreducible representations of a simple three-dimensional Lie algebra over a field of finite characteristic, *Math. Notes Acad. Sci. USSR* **2** (1967), 760–767, (russ. orig.:) Неприводимые представления простой трехмерной алгебры Ли над полем конечной характеристики, Матем. заметки **2** (1967), 439–454,

N. Saavedra Rivano
Catégories Tannakiens (Lecture Notes in Mathematics **265**), Berlin etc. 1972 (Springer)

S. P. Smith
Quantum groups. An introduction and survey for ring theorists, pp. 131–178 in: S. Montgomery, L. Small (eds.), *Noncommutative rings,* Proc. Berkeley 1989 (MSRI Publications **24**), New York etc. 1992 (Springer)

R. Steinberg
Lectures on Chevalley Groups, New Haven, Conn. 1968 (Yale Univ.)

M. E. Sweedler
Hopf Algebras, New York 1969 (Benjamin)

T. Tanisaki
Killing forms, Harish-Chandra isomorphisms, and universal R-matrices for quantum algebras, pp. 941–961 in: A. Tsuchiya, T. Eguchi, M. Jimbo (eds.), *Infinite Analysis, Part B,* Proc. Kyoto 1991 (Advanced Series in Mathematical Physics **16**), River Edge, N. J., 1992 (World Scientific)

Notations

We use the standard notations **Z** for the ring of integers, **Q** for the field of rational numbers, and **C** for the field of complex numbers without extra explanation

roman letters

Index

This index contains mainly references to definitions and main results. In some cases (such as "semisimplicity of U–modules") you will be directed to subsections containing results on the topic, but where the phrase from the index does not occur explicitly in that form.

Errata

p. 7, 2nd displayed equation: The numerator should be $v^{a-b} - v^{-a-b} + v^{a+b} - v^{a-b}$

p. 7: Replace the last six lines by:

$$\begin{aligned}\sum_{i=0}^{r}(-1)^i v^{i(r-1)}\begin{bmatrix} r \\ i \end{bmatrix} &= \sum_{i=0}^{r}(-1)^i v^{i(r-1)}\left(v^{-i}\begin{bmatrix} r-1 \\ i \end{bmatrix} + v^{r-i}\begin{bmatrix} r-1 \\ i-1 \end{bmatrix}\right) \\ &= \sum_{i=0}^{r-1}(-1)^i v^{i((r-1)-1)}\begin{bmatrix} r-1 \\ i \end{bmatrix} \\ &\qquad - \sum_{j=0}^{r-1}(-1)^j v^{(j+1)(r-1)} v^{r-j-1}\begin{bmatrix} r-1 \\ j \end{bmatrix}\end{aligned}$$

where we interpret $\begin{bmatrix} r-1 \\ -1 \end{bmatrix}$ as 0. The first sum is equal to 0 by induction, the second one is equal to

$$v^{2r-2}\sum_{j=0}^{r-1}(-1)^j v^{j((r-1)-1)}\begin{bmatrix} r-1 \\ j \end{bmatrix},$$

hence also 0 by induction.

p. 14: The first (three line) display in "1.5:" should end with $Z^{n-1}X^r$ [not $Z^{n+1}X^r$]

p. 24, l. −8: replace 2,12 by 2.12

p. 25, l. −7: replace 2.11 by 2.12

p. 26, lines 1, 2, and 4: replace p by l

p. 26, line −4: replace U^0 by U_0

p. 27, Proof of 2.18, line 2: Lemma 2.17, not 2.18

p. 29, l. −1: In the summand for $i = 0$ the product over j should run from $j = 1$ to $j = r - i - 1 = r - 1$.

p. 30, first displayed line in 2.7: $\cdots = -(q - q^{-1})^{-1}(K - K^{-1})E$.

p. 30, last displayed line in 2.7: $= \dfrac{(K - K^{-1})(q - q^{-1})}{(q - q^{-1})^2} E$.

p. 32, l. 1: replace "$E \otimes K^{-1} + 1 \otimes F$" by "$E \otimes 1 + K \otimes E$"

p. 42, l. -6: replace 3.2(4) by 3.1(4)

p. 42, l. -2: replace 3.2(3) by 3.1(3)

p. 47, l. 1: replace 0.1(2) by 0.2(1)

p. 48, two lines before 3.10(7): $\dots = q^{-r(r-1)}F(KF^rK^{-1})K^{r+1}$

p. 58, lines -1 and -2: replace K_A^{-1} by $K_{\mathrm{wt}A}^{-1}$

p. 60, in 4.14(6): replace μ by ν

p. 61, proof, line 7: formula

p. 64, in 4.18(7): $\dots = q_\alpha^{i(1-r)}E_\beta$.

p. 66, two lines before 4.21(4): the left hand sides (not "right")

p. 67, lines 6–8: The sentence beginning "Theorem 4.21.a and ..." should be reformulated as follows: "The description of the bases of $\widetilde{U}^{\geq 0}$ and $\widetilde{U}^{\leq 0}$ in 4.17 implies the multiplication induces surjective linear maps $U^0 \otimes U^+ \to U^{\geq 0}$ and $U^- \otimes U^0 \to U^{\leq 0}$; Theorem 4.21.a shows that these maps are bijective."

p. 68, l. 1: replace (4) by 4.4(4).

p. 68, l. 10: replace 4.18(1) by 0.2(4).

p. 70, Proof, l. 5: replace $\pm q^a$ by $\pm q_\alpha^a$.

p. 70, l. -3: this equation should be numbered (2).

p. 71, l. -7: replace 3.10(6) by 3.5(2), replace 3.10(7) by 3.10(6).

p. 71, l. -6: replace 3.10(8) by 3.10(9).

p. 72, 5.4, l. -1: replace $M(q^{-a-2})$ by $M(q_\alpha^{-a-2})$.

p. 74, l. -7: replace I' by I.

p. 75, l. 1 of proof of 5.9: replace 5.5(1) by 5.5(4).

p. 77: the fourth line after (2) should begin "for all j such that ...", (i.e, delete "all $\alpha \in \Pi$ and")

p. 77, three lines before (3): the index of $({}^\omega\widetilde{L}(\lambda'))$ should be $-\lambda' + \nu_0$, not $\lambda' - \nu_0$

p. 79, one and three lines after 5.12(7): replace $[a+r]$ by $[a+s]$

p. 79, 5.12(8): The factor after $[K_\alpha; 0]$ should be $F_{\beta_{i+1}}$

p. 80, line after (5): replace 5.1(2) by 5.1(3).

p. 81, 5.14(3): $I = (\beta_1, \beta_2, \dots, \beta_r)$.

p. 81, l. $-10/-9$ ... homomorphism of algebras $\mathbf{Q}[v, v^{-1}] \to k$ that takes v ...

p. 83, l. 9: a homomorphic image of $M(\mu)$ [not of $M(\lambda)$]

p. 83, l. 10: replace 5.6.b by 5.4.b

p. 84, l. −6: replace 5.19(1) by 5.18

p. 84: Formula (3) in 5.19 should be numbered (2). And the direct sum should be over $\nu \leq \lambda - \mu$.

p. 89: The alignment in (2) should be fixed.

p. 93, l. 4: add a ")" at the end.

p. 102, in 5A.8(6): The direct sum should start with $i = 0$ [not: $i = 1$]

p. 102, in 5A.8(6): The direct sum should start with $i = 0$ [not: $i = 1$]

p. 109, l. 7: replace 5.2(1) by 5.2(2).

p. 111, 6.8, l. −2: replace (2) by (1).

p. 112, l. 2 efter (1): replace 5.1(2) by 5.1(1).

p. 113, in 6.10(8): Replace F_J by E_J in the middle of the first line

p. 116, l. 1: replace K_α by $K_{\mu+\mu'-\alpha}$

p. 117, l. 5: replace 6.14(4) by 6.14(3)

p. 117, l. −1: replace 6.10(3) by 6.13(1)

p. 122, Proof, l. 2: replace 6.19(2) by 6.20(2)

p. 131, l. 3 of 7.4: replace "by Lemma 4.14" by "by 4.13(1)"

p. 134, two lines before (4): replace the second v_{i+1} by v_{i+2}

p. 138, in 7.12(2): Replace R^{rs}_{hl} (on the left hand side of the equation) by R^{rs}_{lh}

p. 138, l. 4 of proof of 7.12: $\dots = \sum_{h,l,r,s} R^{rs}_{lh}(c'_{li}c_{hj})(u)e_r \otimes e'_s$

p. 148, l. −6: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-j(i+1)}$

p. 148, l. −5: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-(j-1)(i+1)}$

p. 148, l. −1: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-ji}$

p. 149, l. 7 and l. 9: replace $a(m-j+1)$ by $a(m-j-1)$

p. 150, l. 3/4 after the first display: delete "Proposition"

p. 152, l. 4: $= (E_\alpha u' - K_\alpha u K_\alpha^{-1} E_\alpha) T_\alpha(v)$

p. 161, l. 2 of proof of Lemma 8.21: insert $\sum_{i=0}^{r}$ after the last "="

p. 161, l. −5 of 8.21: E^i_α [not $E^{(i)}_\alpha$]

p. 162, l. 7 of 8.23: $E^a_\beta E^b_\alpha$ resp. of all $E^a_\alpha E^b_\beta \dots$ [no parentheses in the exponents]

p. 164, l. 7 after (5): $q^{-(\gamma_i,\gamma_j)}$ [replace β by γ and note the sign!]

p. 167, l. 1: replace 8,26 by 8.26

p. 191, l. 2 of the proof of b): description

p. 191, in 9.6(2): It is more logical to write $\widetilde{F}_\alpha x_{\mu-\alpha} = F_\alpha^{(2)} x_\mu = 0$.

p. 193, in 9.7(2): replace $F_\alpha^{(j)}$ by $F_\alpha^{(j-1)}$

p. 194, l. −6: replace $\mathcal{B}$ by $\mathcal{B}(\lambda)$.

p. 195, l. 10: replace v_s by v_r

p. 195, first display: both direct sums should be $\bigoplus_{i=1}^{r}$

p. 197, l. 13: $\mathcal{N} = \mathcal{M} \cap N$ [not $= \mathcal{N} \cap N$]

p. 197, l. 16: ...with $\varphi_i(v_\lambda) = v_i$ ... [not: $\varphi(v_\lambda)$]

p. 199, l. 1: replace $(\mathcal{M}, \mathcal{L})$ by $(\mathcal{M}, \mathcal{B})$

p. 200, l. 2 of Lemma 9.14: replace U_α by U^α

p. 202, l. 1: for all integers $a \geq 0$ and $b > 0$

p. 202, l. 7: this yields one inclusion in (6) [not in (10)]

p. 202, paragraph beginning with "It is now easy to get (1)–(4)": All occurrences of y, z_0 and z_1 should be overlined. (This was forgotten once for y and three times each for z_0 and z_1.)

p. 203, l. −9: replace $r \geq 0$ by $r > 0$

p. 203, 9.16(3): Left hand side should be $\widetilde{E}_\alpha^r(x \otimes b) = 0$

p. 203, 9.16(5): replace "if $i > 0$" by "if $0 < i \leq n+1$"

p. 204, six lines after (6): replace $\widetilde{E}_\alpha^2(x \otimes b) = 0$ by $\widetilde{E}_\alpha^2(x \otimes b) \neq 0$ [This is the line where we get $v \otimes \widetilde{E}_\alpha(x \otimes b)$.]

p. 205, l. 10: replace $\widetilde{E}_\alpha^j b_0$ by $\widetilde{F}_\alpha^j b_0$

p. 205, l. −3: replace 9.16 by 9.15

p. 206, l. −2 of 9.18: $\widetilde{E}_\alpha^{m+1} b' = 0$ [not $\widetilde{E}_\alpha^m b = 0$]

p. 206, l. −1 of 9.18: $= \langle \lambda, \alpha^\vee \rangle$ [not $= \langle \lambda, \alpha^\vee \rangle + 1$]

p. 208, lines −2 and −3 in paragraph beginning "c)": $\dim M > 0$ [not $\dim M < 0$]

p. 208, line −2 in paragraph beginning "c)": " ... if and only if ... " [second 'if' is missing]

p. 208, l. −16: $M^* = \bigoplus_\nu (M_\nu)^*$ [not: $M = \dots$]

p. 208, l. −10: $= \psi(v)(\tau_1(u)v') =$ [not $= \psi(v)(v, \tau_1(u)v') =$]

p. 212, l. −16: this implies $\overline{q^r x} = 0$ [not $\overline{q^r y} = 0$]

p. 214, l. 7: and Lemma 9.18 [not Lemma 9.17]

p. 221, first line of 10.5: …for all $a \in \mathbf{Z}, a > 0$ [i.e., add $a > 0$]

p. 221, line after 10.5(2): and for all $a, n \in \mathbf{Z}$ with $0 \le n \le a$

p. 222: Lemma 10.5 is false as stated. In order to correct it, take $x \in L(\lambda)_\mu$; we should then assume that $\langle \mu, \alpha^\vee \rangle \neq m-2$ in order for the congruence $\widetilde{E}_\alpha F_\alpha^{(m-1)} x \equiv F_\alpha^{(m-2)} x$ in (5) to hold. Similarly, for the second congruence in 10.5(9) on p. 223 to hold, we need that $\langle \mu, \alpha^\vee \rangle \ge m - i$ or $\langle \mu, \alpha^\vee \rangle < m - 2i$. The mistake in the proof occurs two lines after 10.5(8) on p. 223: The equality $\widetilde{E}_\alpha F_\alpha^{(m-1)} x_0 = F_\alpha^{(m-2)} x_0$ holds only if $\langle \mu, \alpha^\vee \rangle \ge m - i$ or $\langle \mu, \alpha^\vee \rangle < m - 2i$ (in the second case because both sides are 0). — The correction makes necessary a small change in the proof of Lemma 10.6. We apply there Lemma 10.5 with $x = yv_\lambda \in L(\lambda)_{\lambda-\nu+n\alpha}$ and $m = n+1$. So we need to know that $\langle \lambda - \nu + n\alpha, \alpha^\vee \rangle \neq n-1$, i.e., that $\langle \lambda, \alpha^\vee \rangle \neq \langle \nu, \alpha^\vee \rangle - (n+1)$. That is no problem since we later assume that $\langle \lambda, \alpha^\vee \rangle \ge \langle \nu, \alpha^\vee \rangle + n$. So we just have to state that assumption somewhat earlier on in the proof.

p. 222, l. −1: $\begin{bmatrix} m-i+j \\ j \end{bmatrix}_\alpha$ [j, not i, in second row]

p. 224, two lines before (4): $q_\alpha^{i(\langle \lambda-\nu+n\alpha, \alpha^\vee \rangle + i + 1)}$ in the numerator [additional n]

p. 224, one line before (4): $\lambda - \nu + (i+n)\alpha$

p. 224, 10.6(4): $q_\alpha^{i(2\langle \lambda-\nu, \alpha^\vee \rangle + 2i + 3n)}$ in the numerator [$3n$ instead of $4 - n$]

p. 224, last display in 10.6: first factor should be $q_\alpha^{i(2\langle \lambda-\nu, \alpha^\vee \rangle + 2i + 3n)}$ [as in previous correction], the middle factor on the right hand side should be $q_\alpha^{i(\langle \lambda, \alpha^\vee \rangle - \langle \nu, \alpha^\vee \rangle + 2i + 4n + 1)}$

p. 224, l. −5: …made at the end …[delete 'in']

p. 224, l. −3: replace Proposition by Theorem

p. 228, l. 8 replace Lemma 10.9 by Proposition 10.9

p. 229, first line of Remark: $(\mathcal{L}(\infty), \mathcal{B}(\infty))$

p. 230, Prop. 10.14: "$\overline{\varphi}_\lambda$ induces" [NOT: $\overline{\varphi}_\lambda(b)$]

p. 231, 10.16(1): right hand side is $U_\nu^+ K_\nu$

p. 232, l. −3: replace 6.16(6) by 6.15(6)

p. 233, l. −3 of 10.17: replace $r_\alpha(x)$ by $r'_\alpha(x)$

p. 238, 11.1(5): Add an index α to the Gaussian binomial coefficient

p. 238, 11.1(6): Replace both μ by ν

p. 239, Lemma 11.2.b: replace *with* $r'_\alpha(u) = 0$ *for all* n by *with* $r'_\alpha(u_n) = 0$ *for all* n

p. 245, first line after 11.8(2): Replace $U^0_{\mathbf{Z}}$ by $U^-_{\mathbf{Z}}$

p. 245, Proposition 11.9: The word 'Proposition' is in a wrong font

p. 247, l. −2 in 11.10: add a ")" after $q\mathcal{L}(\infty)_{-\nu+n\alpha}$

p. 249, three lines after 11.12(2): 11.10.c [not 11.10.b]

p. 250, two lines before 11.13(4): $e_\alpha(d) = 0$ [not > 0]

p. 251, l. 3: $f_\alpha(\overline{\varphi}_\lambda(b))$ [not (d)]

p. 251, first line after 11.13(6): $S = \{G^\alpha(b)v_\lambda \mid b \in \mathcal{B}(\infty)_{-\nu}, \overline{\varphi}_\lambda(b) \neq 0, e_\alpha(b) \geq n\}$

p. 251, first line after 11.14(1): $\varphi_\lambda(b) = \widetilde{F}_{\alpha_1} \dots \widetilde{F}_{\alpha_{r-1}} \widetilde{F}_\beta \varphi_\lambda(1) = 0$

p. 252, l. 12: …yields 11.10.b.

p. 252, two lines before 11.15(1): $S = \{G(b)v_\lambda \mid b \in \mathcal{B}(\infty)_{-\nu}, \overline{\varphi}_\lambda(b) \neq 0\}$

p. 254, l. −3: $= F_\alpha^{(r-i)} F_{\alpha+\beta}^{(i)} F_\beta^{(s-i)} + \dots$ AND: add an index α to the Gaussian binomial coefficient

p. 255, l. 3: $\equiv F_\alpha^{(r-i)} F_{\alpha+\beta}^{(i)} F_\beta^{(s-i)}$

p. 255, l. 5: $\equiv F_\alpha^{(r-i)} F_\beta^{(s)} F_\alpha^{(i)}$

p. 255, l. 6: So we see that $F_\alpha^{(r-i)} F_\beta^{(s)} F_\alpha^{(i)} \dots$

p. 255, Proposition 11.18: $\tau G(b) = G(\tau b)$

p. 255, line 2 of the proof of 11.18: $\tau G(b) = G(\tau b)$

p. 255, line 7 of the proof of 11.18: $r'_\alpha(u'_j) = 0$

p. 255, l. −8: since $e_\alpha(b') = 0$ [NOT: > 0]

p. 257, four lines before 11.19(3): $(U^-_{\mathbf{Z}})_{-\mu} \otimes \mathbf{C}$

p. 260, ref. Jimbo 2: replace "q–difference analogue" by "q–analogue"

p. 261, ref. Rudakov & Shafarevich: Delete the comma at the end

Last Update: 30 January 2009

The material is very well motivated ... Of the various monographs available on quantum groups, this one ... seems the most suitable for most mathematicians new to the subject ... will also be appreciated by a lot of those with considerably more experience.

—***Bulletin of the London Mathematical Society***

Since its origin, the theory of quantum groups has become one of the most fascinating topics of modern mathematics, with numerous applications to several sometimes rather disparate areas, including low-dimensional topology and mathematical physics. This book is one of the first expositions that is specifically directed to students who have no previous knowledge of the subject. The only prerequisite, in addition to standard linear algebra, is some acquaintance with the classical theory of complex semisimple Lie algebras. Starting with the quantum analog of $\mathfrak{sl}_2$, the author carefully leads the reader through all the details necessary for full understanding of the subject, particularly emphasizing similarities and differences with the classical theory. The final chapters of the book describe the Kashiwara–Lusztig theory of so-called crystal (or canonical) bases in representations of complex semisimple Lie algebras. The choice of the topics and the style of exposition make Jantzen's book an excellent textbook for a one-semester course on quantum groups.

ISBN 978-0-8218-0478-0

9 780821 804780

GSM/6