Principles of Functional Analysis

SECOND EDITION

Martin Schechter

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Abstract. The book is intended for a one-year course for beginning graduate or senior undergraduate students. However, it can be used at any level where the students have the prerequisites mentioned below. Because of the crucial role played by functional analysis in the applied sciences as well as in mathematics, the author attempted to make this book accessible to as wide a spectrum of beginning students as possible. Much of the book can be understood by a student having taken a course in advanced calculus. However, in several chapters an elementary knowledge of functions of a complex variable is required. These include Chapters 6, 9, and 11. Only rudimentary topological or algebraic concepts are used. They are introduced and proved as needed. No measure theory is employed or mentioned.

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$BS''D$

To my wife, children, and grandchildren.

May they enjoy many happy years.

Contents

PREFACE TO THE REVISED EDITION

The first edition of *Principles of Functional Analysis* enjoyed a successful run of 28 years. In revising the text, we were confronted with a dilemma. On the one hand, we wanted to incorporate many new developments, but on the other, we did not want to smother the original flavor of the book. As one usually does under such circumstances, we settled for a compromise. We considered only new material related to the original topics or material that can be developed by means of techniques existing within the original framework. In particular, we restricted ourselves to normed vector spaces and linear operators acting between them. (Other topics will have to wait for further volumes.) Moreover, we have chosen topics not readily available in other texts.

We added sections to Chapters 3, 5, 7, 9, and 13 and inserted a new chapter – Chapter 14. (The old Chapter 14 now becomes Chapter 15.) Added topics include products of operators (Sections 5.7 and 7.7), a more general theory of semi-Fredholm operators (Sections 5.6 and 7.6), Riesz operators (Section 9.6), Fredholm and semi-Fredholm perturbations (Sections 9.6 and 9.7), spectral theory for unbounded selfadjoint operators (Section 13.6), and measures of operators and perturbation functions (Chapter 14).

We attempted to strengthen those areas in the book that demonstrate its unique character. In particular, new material introduced concerning Fredholm and semi-Fredholm operators requires minimal effort since the required machinery is already in place. By these means we were able to provide very useful information while keeping within our guidelines.

The new chapter (Chapter 14) deserves some additional remarks. It is designed to show the student how methods that were already mastered can be used to attack new problems. We gathered several topics which are new, but relate only to those concepts and methods emanating from other parts of the book. These topics include perturbation classes, measures of noncompactness, strictly singular operators and operator constants. This last topic illustrates in a very surprising way how a constant associated with an operator can reveal a great deal of information concerning the operator. No new methods of proof are needed, and, again, most of this material cannot be readily found in other books.

We went through the entire text with a fine toothed comb. The presentation was clarified and simplified whenever necessary, and misprints were corrected. Existing lemmas, theorems, corollaries and proofs were expanded when more elaboration was deemed beneficial. New lemmas, theorems and corollaries (with proofs) were introduced as well. Many new problems were added.

We have included two appendices. The first gives the definitions of important terms and symbols used throughout the book. The second lists major theorems and indicates the pages on which they can be found.

The author would like to thank Richard Jasiewicz for installing LaTeX 2_ε into his computer. He would also like to thank the editors and staff of the AMS for helpful suggestions.

Irvine, California

March, 2001

$TVSLB''O$

The following are a few excerpts from a review of the original edition by Einar Hille in the *American Scientist.*[1]

" 'Charming' is a word that seldom comes to the mind of a science reviewer, but if he is charmed by a treatise, why not say so? I am charmed by this book."

"Professor Schechter has written an elegant introduction to functional analysis including related parts of the theory of integral equations. It is easy to read and is full of important applications. He presupposes very little background beyond advanced calculus; in particular, the treatment is not burdened by topological 'refinements' which nowadays have a tendency of dominating the picture."

"The book can be warmly recommended to any reader who wants to learn about this subject without being deterred by less relevant introductory matter or scared away by heavy prerequisites."

[1]From Hille, Einar, Review of *Principles of Functional Analysis,* American Scientist {Vol. 60}, No. 3, 1972, 390.

FROM THE PREFACE TO THE FIRST EDITION

Because of the crucial role played by functional analysis in the applied sciences as well as in mathematics, I have attempted to make this book accessible to as wide a spectrum of beginning students as possible. Much of the book can be understood by a student having taken a course in advanced calculus. However, in several chapters an elementary knowledge of functions of a complex variable is required. These include Chapters 6, 9, and 11. Only rudimentary topological or algebraic concepts are used. They are introduced and proved as needed. No measure theory is employed or mentioned.

The book is intended for a one-year course for beginning graduate or senior undergraduate students. However, it can be used at any level where the students have the prerequisites mentioned above.

I have restricted my attention to normed vector spaces and their important examples, Banach and Hilbert spaces. These are venerable institutions upon which every scientist can rely throughout his or her career. They are presently the more important spaces met in daily life. Another consideration is the fact that an abundance of types of spaces can be an extremely confusing situation to a beginner.

I have also included some topics which are not usually found in textbooks on functional analysis. A fairly comprehensive treatment of Fredholm operators is given in Chapters 5 and 7. I consider their study elementary. Moreover, they are natural extensions of operators of the form $I - K$, K compact. They also blend naturally with other topics. Additional topics include unbounded semi-Fredholm operators and the essential spectrum considered in Chapter 7. Hyponormal and seminormal operators are treated in Chapter 11, and the numerical range of an unbounded operator is studied in Chapter 12. The last chapter is devoted to the study of three types of operators on the space $L^2(-\infty, \infty)$.

One will notice that there are few applications given in the book other than those treated in the last chapter. In general, I used as many illustrations as I could without assuming more mathematical knowledge than is needed to follow the text. Moreover, one of the basic philosophies of the book is that the theory of functional analysis is a beautiful subject which can be motivated and studied for its own sake. On the other hand, I have devoted a full chapter to applications that use a minimum of additional knowledge.

The approach of this book differs substantially from that of most other mathematics books. In general one uses a "tree" or "catalog" structure, in which all foundations are developed in the beginning chapters, with later chapters branching out in different directions. Moreover, each topic is introduced in a logical and indexed place, and all the material concerning that topic is discussed there complete with examples, applications, references to the literature and descriptions of related topics not covered. Then one proceeds to the next topic in a carefully planned program. A descriptive introduction to each chapter tells the reader exactly what will be done there. In addition, we are warned when an important theorem is approaching. We are even told which results are of "fundamental importance." There is much to be said for this approach. However, I have embarked upon a different path. After introducing the first topic, I try to follow a trend of thought wherever it may lead without stopping to fill in details. I do not try to describe a subject fully at the place it is introduced. Instead, I continue with my trend of thought until further information is needed. Then I introduce the required concept or theorem and continue with the discussion.

This approach results in a few topics being covered in several places in the book. Thus, the Hahn-Banach theorem is discussed in Chapters 2 and

9, with a complex form given in Chapter 6, and a geometric form in Chapter 7. Another result is that complex Banach spaces are not introduced until Chapter 6, the first place that their advantage is clear to the reader.

This approach has further resulted in a somewhat unique structure for the book. The first three chapters are devoted to normed vector spaces, and the next four to arbitrary Banach spaces. Chapter 8 deals with reflexive Banach spaces, and Chapters 11 – 13 cover Hilbert spaces. Chapters 9 and 10 discuss special topics.

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Index

This excellent book provides an elegant introduction to functional analysis … carefully selected problems … This is a nicely written book of great value for stimulating active work by students. It can be strongly recommended as an undergraduate or graduate text, or as a comprehensive book for self-study.

—***European Mathematical Society Newsletter***

Functional analysis plays a crucial role in the applied sciences as well as in mathematics. It is a beautiful subject that can be motivated and studied for its own sake. In keeping with this basic philosophy, the author has made this introductory text accessible to a wide spectrum of students, including beginning-level graduates and advanced undergraduates. The exposition is inviting, following threads of ideas, describing each as fully as possible, before moving on to a new topic. Supporting material is introduced as appropriate, and only to the degree needed. Some topics are treated more than once, according to the different contexts in which they arise. The prerequisites are minimal, requiring little more than advanced calculus and no measure theory. The text focuses on normed vector spaces and their important examples, Banach spaces and Hilbert spaces. The author also includes topics not usually found in texts on the subject.

This Second Edition incorporates many new developments while not overshadowing the book's original flavor. Areas in the book that demonstrate its unique character have been strengthened. In particular, new material concerning Fredholm and semi-Fredholm operators is introduced, requiring minimal effort as the necessary machinery was already in place. Several new topics are presented, but relate to only those concepts and methods emanating from other parts of the book. These topics include perturbation classes, measures of noncompactness, strictly singular operators, and operator constants. Overall, the presentation has been refined, clarified, and simplified, and many new problems have been added. The book is recommended to advanced undergraduates, graduate students, and pure and applied research mathematicians interested in functional analysis and operator theory.

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