GRADUATE STUDIES IN MATHEMATICS 206

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Preface

Geometric flows are evolution equations at the intersection of differential equations and geometry. The field of study is characterized by the deformation of geometric objects by geometric attributes such as curvature, and the equations that arise are, in an appropriate "gauge", nonlinear parabolic differential equations. These equations have extensive applications to physical and geometric problems arising in industry, materials science, computer vision and image processing, physics, and pure mathematics. The flows may be viewed from the perspectives of geometry and partial differential equations, or perhaps more aptly as a synthesis of both.

Geometric flows come in many flavors — there are, for instance, **extrinsic flows** (e.g., the curve shortening, mean curvature, Lagrangian mean curvature, Gauß curvature, inverse-mean curvature, and Willmore flows), **intrinsic flows** (e.g., the Ricci, Kähler–Ricci, Yamabe, Calabi, Chern–Ricci, cross curvature, and renormalization group flows), **flows of maps** (e.g., the heat equation on Riemannian manifolds, the harmonic map heat flow, and diffeomorphism-preserving flows), **flows of connections** (e.g., the Yang–Mills flow), as well as flows of further geometric structures (e.g., the G_2-flow), and even **discrete curvature flows** (e.g., discrete surface Ricci and discrete Yamabe flows).

Extrinsic flows are characterized by a submanifold evolving in an ambient space with velocity determined by its extrinsic curvature. The goal of this book is to give an extensive introduction to a few of the most prominent extrinsic flows, namely, the curve shortening flow, the mean curvature flow, the Gauß curvature flow, the inverse-mean curvature flow, and fully nonlinear flows of mean curvature and inverse-mean curvature type. We highlight techniques and behaviors that frequently arise in the study of these (and other) flows. To illustrate the broad applicability of the techniques developed, we also consider general classes of fully nonlinear curvature flows. We do not consider "higher-order" flows, such as the Willmore flow.

This book is written at the level of a graduate student who has had a basic course in differential geometry and has some familiarity with partial differential equations. It is intended also to be useful as a reference for specialists. In general, we provide detailed proofs, although for some more specialized results we may present only the main ideas; in such cases, we provide references for complete proofs. The coverage is not comprehensive; rather, selected topics are chosen that establish the foundation for further study. A brief survey of additional topics, with extensive references, can be found in the notes and commentary at the end of each chapter.

A unifying theme throughout these notes is *monotonicity formulae*. Such formulae are fundamental in the qualitative study of partial differential equations, geometric analysis, and geometric evolution equations. The reason is simple: A monotone quantity not only gives some control on a solution but usually also shows that the solution is improving in some sense (unless it is already in equilibrium). On the other hand, for many nonlinear evolution equations, singularities can occur. This is true even in the case of the diffusive, or heat-type, equations that we shall study. Often, the control provided by monotonicity formulae may be bootstrapped to obtain global existence and convergence for certain classes of initial data (such as closed convex hypersurfaces) or to analyze and classify singularities. The classification of singularities is a major aspect of formulating a way to flow past them. In this respect, similarity solutions are central. Similarity solutions are those solutions whose shapes do not change in time. So they do *not* improve; indeed, they are already in some sense the best possible solutions! As we shall see, monotonicity formulae may often be used to show that the shape of singularities approaches that of self-similar solutions.

A Guide for the Reader

The heat equation (Chapter 1)

This part consists of a single chapter, in which we briefly discuss the model for all geometric evolution equations — the classical heat equation on Euclidean space. We investigate its invariance properties and corresponding invariant solutions (a.k.a. "solitons"). We introduce the maximum principle, the Poincaré inequality, Bernstein estimates, the Harnack inequality, and entropy monotonicity formulae, all of which will arise in different contexts in later chapters. We characterize the long-time behavior of periodic solutions and introduce "multipoint maximum principle" methods, another theme which appears repeatedly throughout the book.

Curve shortening flow (Chapters 2–4)

Chapter 2 introduces the curve shortening flow of planar curves. We derive equations for the evolution of geometric quantities such as curvature, length, area, and the support function. We investigate special solutions such as the shrinking circle, the Grim Reaper, and the paperclip. We obtain Bernstein-type derivative estimates for the curvature and use them to characterize the singular time. Short-time existence and uniqueness of solutions are not proved here since they follow from the corresponding results for mean curvature flow proved in Chapter 6.

In Chapter 3, we prove the Gage–Hamilton and Grayson convergence theorems for the curve shortening flow of embedded planar curves. Our approach is to develop successive refinements of maximum principle arguments involving the distance function. The most basic is the avoidance principle, a modification of which implies that embeddedness is preserved under the flow via a chord-arc estimate. A little more work yields Huisken's distance comparison estimate and then the curvature bound by distance comparison by Bryan and the first author. This estimate, combined with straightforward bootstrapping arguments, leads quickly to the convergence result.

Finally, we introduce, in Chapter 4, self-similar and ancient solutions to the curve shortening flow. We discuss the classification by Abresch and Langer, Epstein and Weinstein, and Halldorsson of properly immersed self-similar solutions. We present various monotonicity formulae: Gage's isoperimetric ratio monotonicity, Hamilton's differential Harnack and entropy estimates, and Huisken's monotonicity formula. We examine the hairclip and paperclip ancient solutions and discuss the classication, by Daskalopoulos, Hamilton, and Šešum, of closed, convex ancient solutions. In a later chapter, we present a different route to this classification (which also applies in the noncompact setting).

Mean curvature flow (Chapters 5–14)

In the introductory chapter to this part, Chapter 5, we develop the basic theory of hypersurfaces in Euclidean space, including the fundamental Gauß, Codazzi, and Simons identities and variation formulae for surface areas and enclosed volumes. Our point of view is primarily that of parametrized hypersurfaces, although we also develop the geometry of graphical, level set, starshaped, and convex hypersurfaces. A convex hypersurface may be parametrized by its Gauß map and is characterized by its support function. We develop the "time-dependent" geometry of evolving hypersurfaces in two equivalent ways — via evolving orthonormal frames and pullback bundles — and briefly introduce general curvature flows of hypersurfaces.

In Chapter 6, we begin our discussion of mean curvature flow, which is the gradient flow for the area functional. We briefly discuss its invariance properties and self-similar solutions, including shrinking spheres and cylinders, which appear later in the analysis of singularities. We compute the evolution of various geometric quantities and prove short-time existence and uniqueness of solutions. We introduce the maximum principle for scalar and tensor heat-type equations and use it to prove the avoidance principle and the preservation of embeddedness and to characterize the singular time. Finally, we briefly survey the various notions of "weak solutions" to the mean curvature flow, including measure-theoretic solutions, level-set/viscosity solutions, shadow solutions, piecewise smooth solutions, and flows with surgery.

We then present, in Chapter 7, the important work of Ecker and Huisken on the mean curvature flow of entire graphs. We present their global existence theorem, which provides an entire graphical solution to the mean curvature flow for all positive time from any entire initial datum which is locally Lipschitz continuous. We first develop their local estimates for the second fundamental form and its covariant derivatives.

In Chapter 8, we present Huisken's theorem, which states that the mean curvature flow evolves closed, convex hypersurfaces to round points. We start by proving that pinching of the second fundamental form is preserved under the flow using the tensor maximum principle. The scalar maximum principle shows that a certain measure of the "roundness" of the hypersurface is preserved. A Stampacchia iteration argument, making use of the Michael–Simon Sobolev inequality, is then used to show that the roundness tends to perfection at high curvature scales. Convergence to a round point then follows from estimates for the covariant derivatives of the second fundamental form and straightforward bootstrapping arguments. We finish the chapter with a second proof of Huisken's theorem via a width pinching estimate and a compactness argument. Two further proofs will be presented in later chapters.

Chapter 9 concerns the mean curvature flow of *mean convex* hypersurfaces, that is, hypersurfaces with positive mean curvature. The main interest is singularity formation. To motivate this, we begin with a brief discussion of the neckpinch and degenerate neckpinch singularities. We present a general theory which shows that convex pinching conditions for the second fundamental form are preserved, including a rigidity statement: Nonnegative pinching strictly improves unless the solution splits off a line. We prove the Huisken–Sinestrari "convexity estimate" for mean convex solutions and "cylindrical estimates" for "m-convex" solutions using Stampacchia iteration. In the case of convex solutions, the latter estimate recovers Huisken's roundness estimate. Using a general setup, *all* of these estimates are proved at once, substantially shortening the original proofs. We then prove the Huisken–Sinestrari gradient estimate for the curvature using the maximum principle and develop some of its applications.

In Chapter 10, we present two fundamental monotonicity formulae for the mean curvature flow: Huisken's monotonicity formula and Hamilton's differential Harnack estimate. We first encountered these inequalities in the context of curve shortening flow. We also present important applications.

In Chapter 11, we study singularity formation for mean convex solutions. We consider sequences of space-time points approaching a singularity and the associated "blow-up sequences" and ancient limits. In order to take limits, we need an appropriate compactness theory. We achieve this by making use of the Cheeger–Gromov compactness theorem for sequences of Riemannian manifolds to obtain an analogue for sequences of hypersurfaces, which leads to a suitable compactness result for mean curvature flows. Our first application of the compactness theorem is in the analysis of regions of high curvature; namely, we prove the Huisken–Sinestrari "neck detection" lemma, which shows that a long, thin, round "neck" is necessarily forming at the first singular time for a 2-convex solution. As is typical, this follows by reductio ad absurdum using the compactness theorem (and the convexity estimates proved in Chapter 9). We then use Huisken's monotonicity formula and the compactness theory to prove Brakke's regularity theorem in the case of smooth flows (à la White).

We then address singularity formation. We begin by presenting Hamilton's proof of Huisken's theorem, which involves a compactness argument making use of Huisken's monotonicity formula and Hamilton's Harnack inequality. This argument is then extended to the mean convex setting, providing a classification of singularity models. We show that type-I singularity models are necessarily shrinking cylinders and type-II singularity models are necessarily translating self-similar solutions. We also discuss the notions of tangent flows and limit flows, including some of their established and conjectured properties.

Chapter 12 introduces the noncollapsing theory of W. Sheng and X.-J. Wang, as developed by the first author and others. The crucial objects are the inscribed and exscribed curvatures, which lead to the notions of interior and exterior noncollapsing, respectively. The inscribed and exscribed curvatures are extrema of a certain smooth "2-point" function. Using this function as a barrier, we prove that noncollapsing is preserved under the mean curvature flow. Related to these calculations are Simons-type inequalities for the maximum and minimum principal curvatures as well as the inscribed and exscribed curvatures. Noncollapsing leads to a *fourth* proof of Huisken's theorem which (unlike the previous proofs) also applies to the curve shortening flow, providing another proof of the Gage–Hamilton theorem. A further application of noncollapsing is the local curvature estimate of Haslhofer and Kleiner, which is used to obtain a new proof (by contradiction) of the convexity and cylindrical estimates discussed previously.

Chapter 13 investigates, in greater depth, self-similar and ancient solutions to the mean curvature flow. We first consider shrinking self-similar solutions. After deriving basic equations, we cover the classification by Huisken and Colding–Minicozzi of properly immersed shrinking self-similar solutions. We then present some topological classification results for shrinkers in $\mathbb{R}^3$ — Brendle's theorem that compact embedded genus zero shrinkers in $\mathbb{R}^3$ are round spheres and a result of Mramor and S. Wang which shows that shrinkers in $\mathbb{R}^3$ are unknotted. Next, we investigate translating self-similar solutions. After deriving basic equations, we construct several examples, including the oblique Grim hyperplanes, the bowl soliton, and the translating catenoid. We present a theorem of Spruck and Xiao, which shows that mean convex translators in $\mathbb{R}^3$ are necessarily convex and a theorem of Spruck and Sun which shows that solutions to the translator Dirichlet problem over convex domains are convex. We present rigidity results for the bowl soliton due to X.-J. Wang and R. Haslhofer and state X.-J. Wang's dichotomy (proved in the subsequent chapter): any convex translator which is not entire necessarily lies in a slab region. We construct the "flying wings" of Bourni, the fourth author, and Tinaglia, and the "bowloids" of Hoffman, Martín, Ilmanen, and White. A uniqueness result of Hoffman et al. completes the classification of mean convex translators in $\mathbb{R}^3$.

In Chapter 14, we consider ancient solutions. We present the characterization of the shrinking sphere among closed, convex ancient solutions due to Huisken–Sinestrari and Haslhofer–Hershkovits and prove a convexity estimate for mean convex ancient solutions, which follows a similar argument. We then present a gradient estimate for "entire" ancient solutions due also to Huisken and Sinestrari followed by a classification of the asymptotics of convex ancient solutions. We then present the proof of X.-J. Wang's remarkable dichotomy — every convex ancient solution which is not entire necessarily lies in a stationary slab region — and present some classification results for "ancient ovaloids" (closed, entire examples) and "ancient pancakes" (closed examples in slabs), including a different proof of the characterization of the shrinking sphere mentioned above.

Gauß curvature flows (Chapters 15–17)

We begin our investigation of Gauß curvature flows, in Chapter 15, by analyzing invariance properties and self-similar solutions. We present Chou's characterization of the singular time and the first author's proof of the Firey conjecture. We also discuss the second author's differential Harnack inequality and complete the chapter with an investigation of the variational structures and related entropies associated with the Gauß curvature flows.

In Chapter 16, we study the $\frac{1}{n+2}$-Gauß curvature flow. This flow is exceptional in that it is invariant under the full group of *affine* transformations of $\mathbb{R}^{n+1}$; indeed, it is tangential-diffeomorphism equivalent to the "affine normal flow". As a result, solutions cannot be expected to converge always to "round" points. Instead, we shall see that uniformly convex hypersurfaces converge to *ellipsoids* after appropriately renormalizing. Various methods are used to prove this, including Chou's estimate, the differential Harnack inequality, and the variational structure. However, in contrast to the other convergence results in this book, it is important that we exploit the full affine invariance of the equation.

In Chapter 17, we present the proof of the convergence to a shrinking self-similar solution for flows by powers $\alpha > \frac{1}{n+2}$ of the Gauß curvature due to P. Guan, L. Ni, and the first author, and the classification by Brendle, K. Choi, and Daskalopoulos of these shrinking self-similar solutions (they are round spheres). Combined, these results prove the general Firey conjecture.

Fully nonlinear curvature flows (Chapters 18–20)

We start by introducing, in Chapter 18, a general notion of evolution by curvature and deduce some general properties such as short-time existence of solutions and the avoidance principle, differential Harnack inequalities, entropy monotonicity formulae, and the Alexandrov reflection method.

We then consider, in Chapter 19, contracting flows by speeds that are 1-homogeneous in the principal curvatures, which tend to share phenomena with the mean curvature flow. Indeed, we show that, under certain structure conditions, convex hypersurfaces shrink to round points. We also discuss certain convexity, cylindrical, and noncollapsing estimates.

In Chapter 20 we discuss expanding curvature flows by speeds that are 1-homogeneous in the principal radii. We show that for a large class of such flows of compact convex hypersurfaces, solutions exist for infinite time, expand to infinity, and converge after rescaling to round spheres.

Throughout the book, we have included photos of a cross section of mathematicians who have made important contributions in the field of extrinsic geometric flows. The inclusion of these photos is in the spirit of encouragement and inspiration. We hope that they will enliven the results and make the book more interesting to read.[1]

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[1]Results discussed in this book are often multiauthored, whereas the photos usually appear individually and in different parts of the book, reflecting the fact that many authors have multiple contributions.

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Suggested Course Outlines

This book offers multiple pathways for a one- or two-semester course for students familiar with differential geometry of Euclidean hypersurfaces and some parabolic PDE.

Chapters 1 and 5 present background material (on the heat equation and hypersurface geometry, respectively) and can thus be omitted if not required.

(1) CURVE SHORTENING FLOW: Chapters 1 through 4 would be suitable for an undergraduate course on curve shortening flow.

(2) INTRODUCTION TO GEOMETRIC FLOWS: A selection of material from Chapters 1–3, 5, and 6, Sections 8.1, 8.2–8.4, Chapters 15–16, Section 19.1, and Chapter 20 would be suitable for a broad introduction to extrinsic geometric flows. Chapter 3 can be substituted with Sections 4.1–4.3. Sections 8.2–8.4 can be substituted with Section 8.6, or with Chapter 10 and Sections 11.1 and 11.4.

(3) MEAN CURVATURE FLOW: Chapters 5–12 and selected topics from Chapters 13 and 14 would be suitable for a course on mean curvature flow.

(4) GAUß CURVATURE FLOW: Chapters 2–4 and 15–17 would be suitable for a course on Gauß curvature flow.

(5) SELF-SIMILAR AND ANCIENT SOLUTIONS: An appropriate selection of material from Chapters 1, 2, 4, 6, 10, 11, and 13–17 would be suitable for a course on self-similar and ancient solutions to geometric flows.

Notation and Symbols

$\doteqdot$	defined to be equal to
$\times$	multiplication, if a formula does not fit on one line
$\cdot$	dot product or multiplication
$\otimes$	tensor product
$\odot$	symmetrized tensor product
$\cdot^{\top}$	tangential projection
$\cdot^{\perp}$	normal projection
∂_i	i-th coordinate basis element or partial derivative with respect to x^i: $\frac{\partial}{\partial x^i}$
∂_t	canonical vector field on $M^n \times (\alpha, \omega)$ or partial derivative with respect to "time", t: $\frac{\partial}{\partial t}$
∇	covariant derivative or gradient
∇_a	covariant derivative as directional derivative of functions on the frame bundle
∇_t	covariant time derivative (abstract or as a vector field on the frame bundle)
$\overline{\nabla}$	covariant derivative or gradient with respect to the standard metric on S^n
$\nabla\nabla$ or ∇^2	(covariant) Hessian
Δ	Laplacian (rough Laplacian when acting on tensors)
Δ_f	f-Laplacian
$\langle \cdot, \cdot \rangle$	Riemannian metric or inner product
$\partial\Omega$	boundary of a domain Ω
$\alpha^{\natural}$	dual vector field to the 1-form α

A	area enclosed by a plane curve
$\mathrm{A}_p = d\mathrm{G}\,\|_p$	shape operator
$A_\#$	least shadow area
Area	area of a surface or volume of a hypersurface
$B_r(p)$	extrinsic ball of radius r centered at p
$\mathbb{C}$	set of complex numbers
$C^\infty(M)$	space of smooth functions on M
$C_c^\infty(M)$	space of C^∞ functions on M with compact support
$\mathcal{C}_m^n(R,\alpha)$	the class of $(m+1)$-convex hypersurfaces of $\mathbb{R}^{n+1}$ satisfying certain geometric bounds
const.	constant
d	distance
D	Euclidean covariant derivative
${}^X D$	pullback of Euclidean connection along X
dA	area form of a boundary
$d\mu$	Riemannian area element
dm	weighted area $\mathrm{e}^{-f}\,d\mu$
ds	arc length element
dt	canonical 1-form on $M^n \times (\alpha,\omega)$
diam	diameter
div	divergence
e	Euler's number
$\{e_i\}$	standard Euclidean basis
$E_r(x,t)$	heat ball of radius r based at (x,t)
End	bundle of selfadjoint endomorphisms of (TM,g)
exp	exponential map or exponential function
$\mathcal{F}$	entropy or Perelman's energy functional
$F(M)$	frame bundle of M
$\Gamma(E)$	space of smooth sections of E
Γ_{ij}^k	Christoffel symbols
G	Gauß map
g	Riemannian metric, including pullback metric
$g\,(t)$	time-dependent metric
$\bar{g}$	standard metric on S^n
H, $\vec{H}$	mean curvature, mean curvature vector
H_m	m-th mean curvature

H_{-1}	harmonic mean curvature
$\mathscr{H}^n$	n-dimensional Hausdorff measure
I	a “time” interval
I or g	first fundamental form or pullback metric
II or h	second fundamental form or its pullback
$\vec{\mathrm{II}}$ or $\vec{h}$	vector second fundamental form or its pullback
id	identity
inj	injectivity radius
int or $^\circ$	interior
J	counterclockwise rotation by $\pi/2$
κ	curvature of a plane curve
κ_i	principal curvature of a hypersurface
κ_1, κ_n	smallest, largest principal curvature
$\vec{\kappa}$	n-tuple of principal curvatures
K	Gauß curvature
k	extrinsic ball curvature
$\overline{k}$, $\underline{k}$	inscribed, exscribed curvature
L	various linear operators
L	length of a plane curve or Weingarten map
$\vec{\mathrm{L}}$	normal vector-valued Weingarten tensor
$\mathcal{L}$	Lie derivative or the operator $\alpha K^\alpha(\mathrm{II}^{-1})^{ij}\nabla_i\nabla_j$
log	natural logarithm
M^n	n-manifold (domain of a parametrized hypersurface $X : M^n \to \mathbb{R}^{n+1}$)
$\mathcal{M}^n$	immersed n-manifold (image of a parametrized hypersurface $X : M^n \to \mathbb{R}^{n+1}$)
μ	Riemannian measure
$\widetilde{M}$	universal covering of M
N, $\vec{n}$, ν	unit outward normal
MW	mean width
$\mathbb{N}$	the (positive) natural numbers
$N\mathcal{M}$, NM	normal bundle of $\mathcal{M}$, M
$\mathfrak{N}$, NM	normal bundle of $M^n \times (\alpha, \omega)$
$n\omega_n$ or σ_{n-1}	volume of the unit Euclidean $(n-1)$-sphere
ω_n	volume of the unit Euclidean n-ball
ODE	ordinary differential equation(s)

$OF(M)$	orthonormal frame bundle of M
OU	Ornstein–Uhlenbeck operator
ρ_y	reflection of S^n in the y-direction
ρ_Y	left action of $\mathrm{GL}(n,\mathbb{R})$ on $F(M)$
$P_r(x,t)$	parabolic ball of radius r based at (x,t)
PDE	partial differential equation(s)
$\mathbb{R}$ $(\mathbb{R}_+)$	the set of (positive) real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
r_i	principal radius of a hypersurface
R, Rc, Rm	scalar, Ricci, and Riemann curvature tensors
σ	support function
$\mathfrak{S}$, TM	spatial tangent bundle of $M \times (\alpha, \omega)$
S^n	unit radius n-dimensional sphere
$\mathrm{S}(n)$ or $\mathrm{S}^{n\times n}$	normed linear space of symmetric (real) $n \times n$ matrices (equipped with the Hilbert–Schmidt norm)
S^n_{y+} (S^n_{y-})	upper (lower) hemispheres in the y-direction
$S\mathcal{M}$, SM	unit tangent bundle of $\mathcal{M}$, M
$\mathrm{SO}\,(n)$	(real) special orthogonal group
$\mathfrak{so}(n)$	Lie algebra of $\mathrm{SO}\,(n)$
supp	support of a function
θ	normal angle of a plane curve
T	unit tangent vector to a curve
$T\mathcal{M}$, TM	tangent bundle of $\mathcal{M}$, M
$T^*\mathcal{M}$, T^*M	cotangent bundle of $\mathcal{M}$, M
T^n	n-dimensional torus
tr or trace	trace
Vol	volume enclosed by a hypersurface
w	width function
X	typically a parametrized hypersurface
X_t	parametrized hypersurface at time t
y^j_a	coordinates on the frame bundle
$\mathbb{Z}$	ring of integers

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