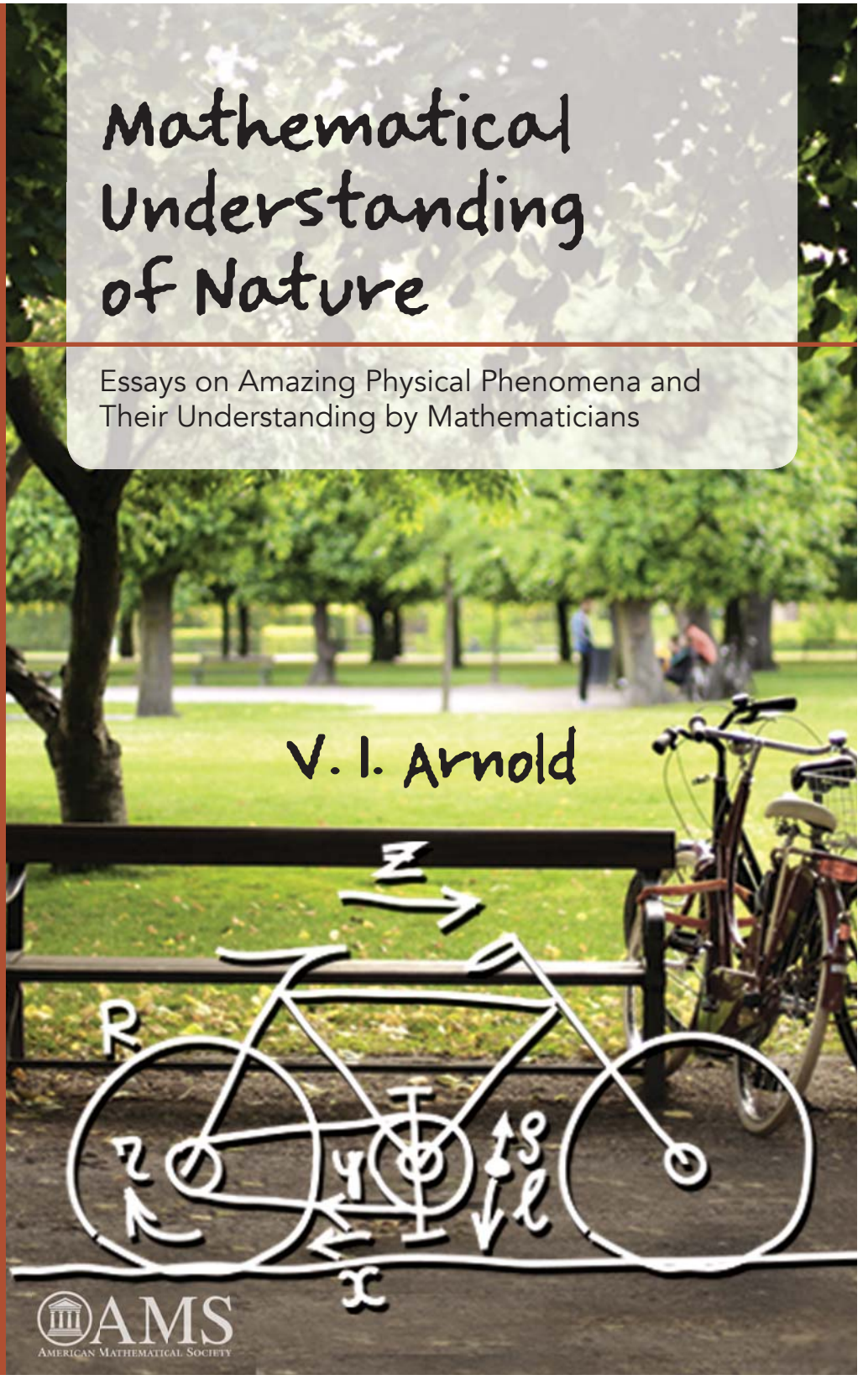
Mathematical Understanding of Nature

Essays on Amazing Physical Phenomena and Their Understanding by Mathematicians



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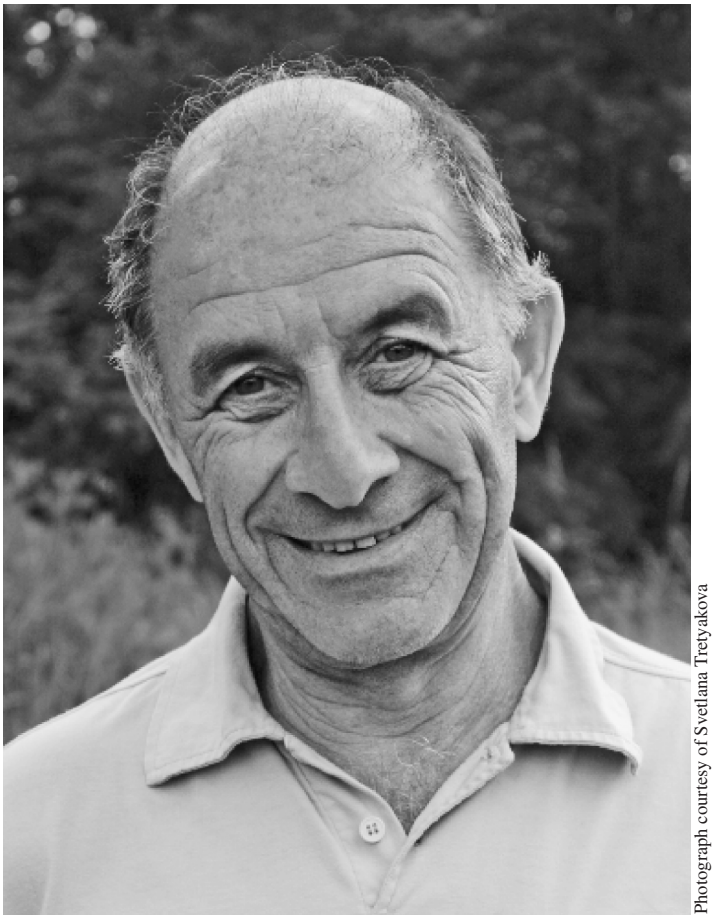

Photograph courtesy of Svetlana Tretyakova

Vladimir Igorevich Arnold
June 12, 1937–June 3, 2010

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Mathematical Understanding of Nature

Essays on Amazing Physical Phenomena and Their Understanding by Mathematicians

V. I. Arnold

Translated by
Alexei Sossinsky
Olga Sipacheva

AMS
American Mathematical Society
Providence, Rhode Island

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Contents

Foreword

At the early age of eleven, the author of this book participated in the "Children Learned Society", organized at home by prominent Russian mathematician and computer scientist, A. A. Lyapunov (the Russian acronym, ДНО, which means "bottom", can be also interpreted as the "Voluntary Learned Society"). In a "Kvant" interview (1990),[1] Arnold remembers:

> The curriculum included mathematics and physics, along with chemistry and biology, including genetics, that was just recently banned (a son of one of our best geneticists was my classmate; in a questionnaire, he wrote: "my mother is a stay-at-home mom, my father is a stay-at-home dad").

Natalia Lyapunova, a daughter of A. A. Lyapunov, recalls:[2]

> ... And look what were the topics of the talks: "The structure of the solar system", "On comets", "Molecular forces"... One cannot forget the talk "Waves" by Dima Arnold. We had a huge dinner table, extendable to 6 sections. The table was unfolded, an aquarium with water was put into

[1] `http://kvant.mccme.ru/1990/07/intervyu_s_viarnoldom.htm`, in Russian.
[2] `http://oso.rcsz.ru/inf/pp/177`, in Russian.

> the hole, and a slide projector was placed underneath. At the time no one had such a projector, but my dad found one somewhere. The light went through the water whose surface projected on the ceiling. Two corks were floating in the aquarium; one needed to give them a push, and the waves started: circular, counter, interference! And all this is projected on the ceiling! Dima is lecturing, and visual demonstrations follow.... I was then in the 4th grade....

The present book is written in the spirit of the "Children Learned Society", and its target audience consists of "young mathematicians of all ages".[3]

The level of sophistication of these essays differs substantially, from being accessible to a high school student to presenting serious challenges for a seasoned researcher. In my opinion, this is a merit of the book: it belongs, equally well, to a high school library and to a faculty lounge.

The philosophy of the author is clearly visible:

> Mathematics is part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap.[4]

A popularizer of mathematics finds himself between a rock and a hard place. According to Michael Faraday (one of the greatest popularizers of science),

> Lectures which really teach will never be popular;
> lectures which are popular will never teach.

The present book is a (rare) counter-example to Faraday's maxim: it is eye-opening, open-ended, and never boring.

In the preface, Arnold says:

[3] In his memories of Vladimir Rokhlin, Arnold quotes from Courant: "... a mathematician should be considered young for as long as he is inclined to discuss math at the most inappropriate times".

[4] V. Arnold. "On teaching mathematics".

> Examples teach no less than rules, and errors more than correct but abstruse proofs.

Indeed, there is an error in the essay "What Force Drives a Bicycle Forward?", and the reader is encouraged to ponder what is going on.[5]

There is another special feature of this book that I have to comment upon, its provocative in-your-face style. Arnold was on a crusade against a formalized approach to mathematics or, in his parlance, "criminal Bourbakization". In this fight, he would take no prisoners– consider, for example, his famous 'mathematical duel' with J-P. Serre on Bourbaki at the Institut Henri Poincaré on March 13, 2001.[6]

Equally passionately, Arnold was fighting against the incorrect attribution of mathematical results. I cannot help but quote from Michael Berry's website:[7]

> Three laws of discovery
>
> 1. Arnold's law (implied by statements in his many letters disputing priority, usually in response to what he sees as neglect of Russian mathematicians):
> *Discoveries are rarely attributed to the correct person.*
> (Of course Arnold's law is self-referential.)
>
> 2. Berry's law (prompted by the observation that the sequence of antecedents under law 1 seems endless):
> *Nothing is ever discovered for the first time.*
>
> 3. Whitehead's law (quoted by Max Dresden at the beginning of his biography of Kramers):
> *Everything of importance has been said before by someone who did not discover it.*

I suspect that Arnold used hyperbole and overstated his opinions on purpose; the reader should be ready to take his most extreme claims with a grain of salt.

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[5] See G. Hart's recent take on this problem at `http://www.simonsfoundation.org/multimedia/mathematical-impressions-multimedia/the-bicycle-pulling-puzzle/`

[6] `http://www.etnoka.fr/qualified/one.tcl?info_id=69919`

[7] `http://michaelberryphysics.wordpress.com/quotations/`

Most of the essays in this little book are quite short; therefore, it is not fitting for this foreword to get any longer. Let me finish with another quotation from Arnold's "Kvant" interview that, in my opinion, well represents both the spirit of this book and of its author:

> The word "Mathematics" means science about truth. It seems to me that modern science (i.e., theoretical physics along with mathematics) is a new religion, a cult of truth, founded by Newton 300 years ago.

Serge Tabachnikov

May 2014

Preface

The investigation of a murder led a movie director (a character of a detective story by Victoriya Tokareva[8]) to the conclusion: "Mathematics is that which can be explained."

The main contribution of mathematics to the natural sciences is not in formal computations (or in other applications of ready-made mathematical achievements), but in the investigation of those non-formal questions where the exact setting of the question (what are we searching for and what specific models must be used) usually constitutes half the matter.

The 39 essays collected below have the same goal: to teach the reader not only to multiply large numbers (which sometimes also has to be done), but to guess about unexpected connections between seemingly unrelated phenomena and facts, at times coming from different branches of the natural and other sciences.

Examples teach no less than rules, and errors, more than correct but abstruse proofs. Looking at the pictures in this book, the reader will understand more than learning by rote dozens of axioms (even together with their consequences about what sea the Volga river falls into and what horses eat).

[8] A Soviet and Russian screenwriter and short story writer.

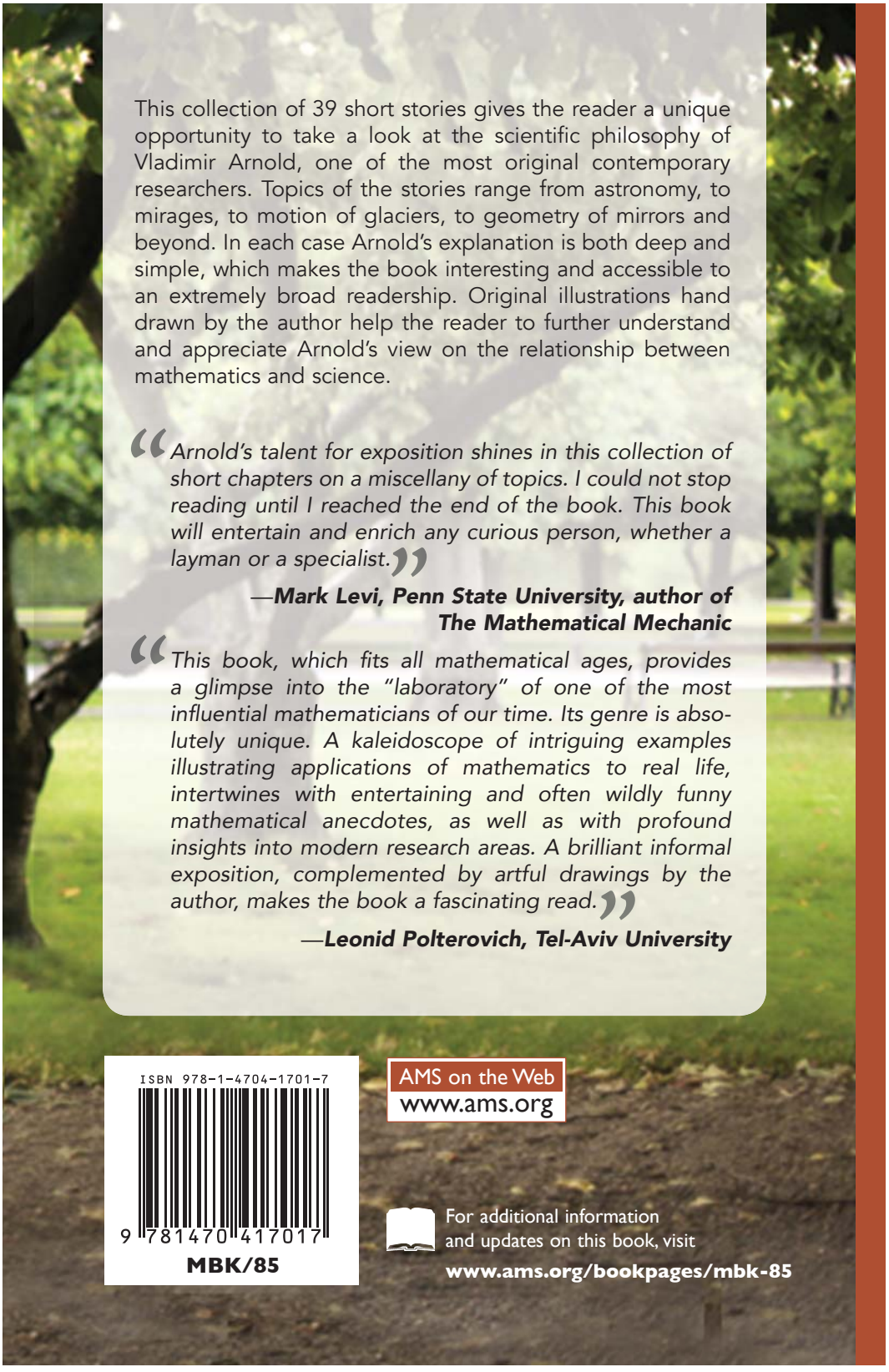
Boris Pasternak wrote that "the question of the usefulness of poetry arises only in periods of its decline, while in periods of its flowering, no one doubts its total uselessness."

Mathematics is not quite poetry, but in it I try to avoid the feeling of decline preached by the enemies of all natural sciences.

Let me also add that Niels Bohr divided true statements into two classes: the trivial ones and those of genius. Specifically, he regarded a true statement as trivial when the opposite statement is obviously false, and a true statement as genius when the opposite statement is just as non-obvious as the original, so that the question of the truth of the opposite statement is interesting and worth studying.

I take this occasion to thank N. N. Andreev who coerced me into writing this book.

From the editors. Vladimir Arnold died on June 3, 2010. He participated in the preparation of the second edition, but did not see the proofs (in which the only changes were in the essays on pages 37–38 and 51–53).



This collection of 39 short stories gives the reader a unique opportunity to take a look at the scientific philosophy of Vladimir Arnold, one of the most original contemporary researchers. Topics of the stories range from astronomy, to mirages, to motion of glaciers, to geometry of mirrors and beyond. In each case Arnold's explanation is both deep and simple, which makes the book interesting and accessible to an extremely broad readership. Original illustrations hand drawn by the author help the reader to further understand and appreciate Arnold's view on the relationship between mathematics and science.

"Arnold's talent for exposition shines in this collection of short chapters on a miscellany of topics. I could not stop reading until I reached the end of the book. This book will entertain and enrich any curious person, whether a layman or a specialist."

—Mark Levi, Penn State University, author of The Mathematical Mechanic

"This book, which fits all mathematical ages, provides a glimpse into the "laboratory" of one of the most influential mathematicians of our time. Its genre is absolutely unique. A kaleidoscope of intriguing examples illustrating applications of mathematics to real life, intertwines with entertaining and often wildly funny mathematical anecdotes, as well as with profound insights into modern research areas. A brilliant informal exposition, complemented by artful drawings by the author, makes the book a fascinating read."

—Leonid Polterovich, Tel-Aviv University

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