

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 85

Lie Groups I

Mikio Ise
Masaru Takeuchi

Lie Groups II

Masaru Takeuchi



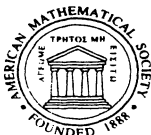
American Mathematical Society

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American Mathematical Society
Providence, Rhode Island

Lie 群 I, II

LIE-GUN I (Theory of Lie Group I)

by Mikio Ise & Masaru Takeuchi

Originally published in Japanese by Iwanami Shoten,
Publishers, Tokyo in 1977 and 1984

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LIE-GUN II (Theory of Lie Group II)

by Masaru Takeuchi

Originally published in Japanese by Iwanami Shoten,
Publishers, Tokyo in 1978

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Translated from the Japanese by Katsumi Nomizu

2000 *Mathematics Subject Classification*. Primary 22-01;
Secondary 22Exx, 17Bxx.

Abstracts and Mathematics Subject Classification numbers can be found on page 2 for Lie Groups I and on page 114 for Lie Groups II.

Library of Congress Cataloging-in-Publication Data

Ise, Mikio.

[Rī-gun. English]

Lie groups I / Mikio Ise, Masaru Takeuchi. Lie groups II / Masaru Takeuchi.

p. cm. — (Translations of mathematical monographs, ISSN 0065-9282; v. 85)

Translation of: Rī-gun.

Includes bibliographical references.

ISBN 0-8218-4544-6 (acid-free)

1. Lie groups I. Takeuchi, Masaru, 1932-. II. Title. III. Title: Lie groups II. IV. Series.

QA387.I7413 1991

512'.55—dc20

90-24683

CIP

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This volume was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}\text{-T}\mathcal{E}\mathcal{X}$,
the American Mathematical Society's $\text{T}\mathcal{E}\mathcal{X}$ macro system.

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ISBN 978-0-8218-4544-8



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