Local Fields and Their Extensions

Second Edition

I. B. Fesenko
S. V. Vostokov

American Mathematical Society
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Local Fields and
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I. B. Fesenko
S. V. Vostokov

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Introduction to the Second Edition

The class of discrete valuation fields appears to be next in significance and order of complexity to the class of finite fields. Among discrete valuation fields a highly important place, both for themselves and in connection with other theories, is occupied by complete discrete valuation fields.

This book is devoted to local fields, i.e. complete discrete valuation fields with perfect residue field.

The time distance between the second edition of “Local Fields and Their Extensions” and its first edition is ten years. During this period, according to Math Reviews, almost one thousand papers on local fields have been published. Some of them have further developed and clarified various topics described in the first edition of this book. On the other hand, the authors of this book have received a variety of useful suggestions and remarks from several dozen readers of the first edition.

All these have naturally led to the second edition of the book.

This book is aimed to serve as an easy exposition of the arithmetical properties of local fields by using explicit and constructive tools and methods. Almost everywhere it does not require more prerequisites than a standard course in Galois theory and a first course in number theory which includes $p$-adic numbers.

The book consists of nine chapters which form the following groups:

- group 1: elementary properties of local fields (Chapter I–III)
- group 2: class field theory for various types of local fields and generalizations (Chapter IV–V)
- group 3: explicit formulas for the Hilbert pairing (Chapter VI–VIII)
- group 4: Milnor $K$-groups of local fields (Chapter IX).

Chapters of the third group were mainly written by S. V. Vostokov and the rest was written by I. B. Fesenko.

The first page of each chapter provides a detailed description of its contents, so here we just emphasize the most important issues and also indicate changes with respect to the first edition.

Chapter I describes the most elementary properties of local fields when one does not look at connections between them, but concentrates on a single field.

Chapter II deals with extensions of discrete valuation fields and already section 1 and 2 introduce a very important class of Henselian fields and describe relations between Henselian and complete fields. We have included more information than in the first edition on ramification subgroups in section 4.
The main object of study in Chapter III is the norm map acting on the multiplicative group and its arithmetical properties. In section 1 we describe its behaviour for cyclic extensions of prime degree. Section 2 shows that almost all cyclic extensions of degree equal to the characteristic of the perfect residue field are generated by roots of Artin–Schreier polynomials. In section 3 we introduce a function which takes into account certain properties of the norm map acting on higher principal units. Our approach to the definition of the Hasse–Herbrand function is different from the approach in other textbooks (where the definition involves ramification groups). Sections 3 and 4 in the second edition now include more applications of our treatment of the Hasse–Herbrand function. Section 5 is devoted to the Fontaine–Wintenberger theory of fields of norms for arithmetically profinite extensions of local fields. This theory links certain infinite extensions of local fields of characteristic zero or \( p \) with local fields of characteristic \( p \). Now the section contains more details on applications of this theory, some of which have been published since 1993.

Chapter IV is on class field theory of local fields with finite residue fields. For this edition we have chosen a slightly different approach from the first edition: for totally ramified extensions we work simultaneously with both the Neukirch map and Hazewinkel homomorphism (which are almost inverse to each other). We hope that this method explains more fully on what is going on behind definitions, constructions and calculations and therefore gives the reader more chances to appreciate the theory. This method is also very useful for applications. Section 1 contains new subsections (1.6)–(1.9) which are required for the study of the reciprocity maps. Sections 2–4 differs significantly from the corresponding parts of the first edition. After proving the main results of local class field theory we review all other approaches to it in the new section 7. The new section 8 presents to the reader a recent noncommutative reciprocity map, which is not a homomorphism but a Galois 1-cycle. This theory is based a generalization of the approach to (abelian) class field theory in this book. We also review results on the absolute Galois group of a local field.

Chapter V studies abelian extensions of local fields with infinite residue field. In the same way as in the first edition, the first three sections discuss in detail class field theory of local fields with quasi-finite residue field. In the new section 4 we describe recent theory of abelian totally ramified \( p \)-extensions of a local field with perfect residue fields of characteristic \( p \) which can be viewed as the largest possible generalization of class field theory of Chapter IV. If a complete discrete valuation field has imperfect residue field, then its class field theory becomes much more difficult. Still, some results on abelian totally ramified \( p \)-extensions of such fields and their norm groups can be established in the framework of this book; we explain some features in the new section 5. The latter also includes a class field theory interpretation of results on some abelian varieties over local fields.

Chapter VI serves as a prerequisite for Chapters VII and VIII. For a finite extension of the field of \( p \)-adic numbers it presents a very useful formal power series method for the study of elements of the fields. The Artin–Hasse–Shafarevich exponential map
is described in section 2 and the Shafarevich basis of the group of principal units in section 5. This Chapter contains many technical results, especially in section 3 and 4, which are of use in Chapter VI.

The aim of Chapter VII is to explain to the reader explicit formulas for the Hilbert symbol. The method is to introduce at first an independent pairing on formal power series and to show that it is well defined and satisfies the Steinberg property (subsection (2.1)). Then a pairing on the multiplicative group of the field induced by the previous pairing is defined. Its properties (independence of a power series presentation and invariance with respect to the choice of a prime element) help one easily show its equality with the Hilbert pairing. The second edition contains many simplifications of the first edition and it also includes more material on interpretations of the explicit formulas and their applications.

Chapter VIII is an exposition of a generalization of the method of Chapter VII to formal groups. The simplest among the groups are Lubin–Tate groups which are introduced in section 1; exercises let the reader see the well known applications of them to local class field theory. Explicit formulas for the generalized Hilbert pairing associated to a Lubin–Tate formal group are presented in section 2. The new section 3 describes a recent generalization to Honda formal groups.

Chapter IX describes the Milnor $K$-groups of fields. Calculations of the Milnor $K$-groups of local fields in section 4 shed a new light on the Hilbert symbol of Chapter IV.

The bibliography includes comments on introductory texts on various applications of local fields.

Numerous remarks and exercises often indicate further important results and theories left outside this introductory book. The most challenging exercises are marked by (○).

Those readers who prefer to start with class field theory of local fields with finite residue fields are recommended to read sections 1–7 of Chapter IV and follow the references to the previous Chapters if necessary.

One of more advanced theories closely related to the material of this book and its presentation is higher local class field theory; for an introduction to higher local fields see [FK].

A reference in Chapter $n$ to an assertion in Chapter $m$ does not state the number $m$ explicitly and only if $m = n$. Briefly on notations: For a field $F$ an algebraic closure of $F$ is denoted by $F^{\text{alg}}$ and the separable closure of $F$ in $F^{\text{alg}}$ is denoted by $F^{\text{sep}}$. Separable and algebraic closures of fields are assumed suitably chosen where it is necessary to make such conventions. $G_F = \text{Gal}(F^{\text{sep}}/F)$ stands for the absolute Galois group of $F$, $\mu_n$ denotes the group of all $n$th roots of unity in $F^{\text{sep}}$.

The text is typed using AMSTeX and a modified version of osudeG style (written by W. Neumann and L. Siebenmann and available from the public domain of Departm of Mathematics of Ohio State University, pub/osutex).

March 2002           I. B. Fesenko           S. V. Vostokov
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**Foreword to the First Edition**

A. Weil was undoubtedly right when he asserted, in the preface to the Russian edition of his book on number theory, that since class field theory pertains to the foundation of mathematics, every mathematician should be as familiar with it as with Galois theory. Moreover, just like Galois theory before it, class field theory was reputed to be very complicated and accessible only to specialists.

Here, however, the parallels between these two theories come to an end. A mathematician who has decided to become acquainted with Galois theory is not confronted with the problem of choosing a suitable exposition: all expositions of it are essentially equivalent, differing only in didactic details. For class field theory, on the other hand, there is a wide range of essentially different expositions, so that it is not immediately obvious even whether the subject is the same.

In the 1960s, it seemed that a universal Galois cohomology approach to class field theory had been found. What is more, the role of homological algebra as a common language unifying various branches of mathematics was becoming clear. Homological algebra could be likened to medieval Latin that served as the means of communication within educated circles. However, just as Latin could not effectively stand up against the originality of individual national languages, so Galois cohomology theory no longer offers the “only reasonable” understanding of class field theory. The goal of the cohomological method was the formation of class fields in which both number and local fields and their arithmetic properties disappear, the whole theory being formalized as a system of axioms. But other expositions of class field theory reveal remarkable properties of number and local fields, that are ignored in the cohomological approach. It has become evident that class field theory is not just an application of cohomology groups, but that it is also closely related with other profound theories such as the theory of formal groups, $K$-theory, etc.

The exposition of this book does not use homological algebra. It presents specific realities of local fields as clear as possible. Despite its limited volume, the book contains a vast amount of information on local fields. It offers the reader the possibility to see the beauty and diversity of this subject.

30 June 1992, Moscow

I. R. Shafarevich
Comment

Introductory sources on related subjects.
local fields [Cas];
algebraic number theory [KKS], [M], [N5], [NSchW], [CF], [FT], [BSh], [Iya], [La2],
[IR], [Ko6], [W];
cyclotomic fields [Wa], [La3];
valuation theory [E], [Rib];
formally \( p \)-adic fields [PR];
non-Archimedean analysis [Kob1–2], [vR], [Schf], [T4], [BGR];
embedding problems [ILF];
formal groups [Fr], [CF], [Iw6], [Haz3];
elliptic curves over number fields [Silv];
local zeta function and Fourier analysis [T1], [RV], [Ig], [Den];
\( p \)-adic \( L \)-functions [Wa], [Iw7], [Hi];
local Langlands correspondence [T7], [Bum], [Kudl], [BaK], [Rit2];
pro-\( p \)-groups [DdSMS], [Wi], [dSSS];
\( p \)-adic Hodge theory [T2], [Fo2], [Sen4,7–9];
\( p \)-adic periods [A];
\( p \)-adic differential equations [RC];
non-Archimedean analytic geometry [Ber];
field arithmetic [FJ], [Jar], [Ef4];
characteristic \( p \) [Gos];
Milnor \( K \)-theory [Bas], [Ro], [Silr], [Gr];
higher local fields and higher local class field theory [FK];
power series over local fields, formal groups, and dynamics [Lu1–2], [Li1–4];
non-Archimedean physics [VVZ], [BF], [Chr], [RTVW], [HS], [Kh].

Symbols and explicit formulas (perfect residue field case). [AH1–2], [Has1–11],
[Sha2], [Kn], [Rot], [Bru1–2], [Henn1–2], [Iw3], [Col1,3], [Wil], [CW1], [dSh1–3],
[Sen3], [Hel], [Des], [Shi], [Sue], [Sh1], [V1–7,9], [Fe1–2], [BeV1–2], [Ab5–6], [Kol],
[Kuz], [Kat6–7], [Ku3–4], [GK], [VG], [DV1–2], [Ben1–2].
Ramification theory of local fields (perfect residue field case). [Kaw1], [Sa], [Tam], [Hei], [Mar1], [Mau1–5], [Mik5–6], [T2], [Wy], [Sen1–2], [ST], [Ep], [KZ], [Fo4], [Win1–4], [Lau1–6], [LS], [CG], [Ab2–4,7–8], [Fe8,11–12].

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## List of Notations

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